The Australian
Curriculum

Subjects	Essential Mathematics, General Mathematics, Mathematical Methods and Specialist Mathematics
Units	Unit 1, Unit 2, Unit 3 and Unit 4
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AUSTRALIAN CURRICULUM, ASSESSMENT AND REPORTING AUTHORITY

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Essential Mathematics

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Essential Mathematics

Rationale and Aims

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring, it has evolved in highly sophisticated and elegant ways to become the language used to describe much of the physical world. Statistics is the study of ways of collecting and extracting information from data and of methods of using that information to describe and make predictions about the behaviour of aspects of the real world, in the face of uncertainty. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Essential Mathematics focuses on enabling students to use mathematics effectively, efficiently and critically to make informed decisions in their daily lives. Essential Mathematics provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts, in a range of workplace, personal, further learning and community settings. This subject offers students the opportunity to prepare for post-school options of employment and further training.

For all content areas of Essential Mathematics, the proficiency strands of understanding, fluency, problem solving and reasoning from the F–10 curriculum are still applicable and should be inherent in students' learning of the subject. Each of these proficiencies is essential, and all are mutually reinforcing. For all content areas, practice allows students to develop fluency in their skills. Students will encounter opportunities for problem solving, such as finding the volume of a solid so that the amount of liquid held in a container can be compared with what is written on the label, or finding the interest on a sum of money to enable comparison between different types of loans. In Essential Mathematics, reasoning includes critically interpreting and analysing information represented through graphs, tables and other statistical representations to make informed decisions. The ability to transfer mathematical skills between contexts is a vital part of learning in this subject. For example, familiarity with the concept of a rate enables students to solve a wide range of practical problems, such as fuel consumption, travel times, interest payments, taxation, and population growth.

The content of the Essential Mathematics subject is designed to be taught within contexts that are relevant to the needs of the particular student cohort. The skills and understandings developed throughout the subject will be further enhanced and reinforced through presentation in an area of interest to the students.

Aims

Essential Mathematics aims to develop students':

- understanding of concepts and techniques drawn from mathematics and statistics
- ability to solve applied problems using concepts and techniques drawn from mathematics and statistics
- reasoning and interpretive skills in mathematical and statistical contexts
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately.

Organisation

Overview of senior secondary Australian Curriculum

ACARA has developed senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications. The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

The senior secondary Australian Curriculum specifies content and achievement standards for each senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) expected of students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment and examinations). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years.

State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into their courses. They will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and Board. They will determine the assessment and certification specifications for their local courses that integrate the Australian Curriculum content and achievement standards and any additional information, guidelines and rules to satisfy local requirements including advice on entry and exit points and credit for completed study.

The senior secondary Australian Curriculum for each subject should not, therefore, be read as a course of study. Rather, it is presented as content and achievement standards for integration into state and territory courses.

Senior secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics, with each subject organised into four units. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.

General Mathematics focuses on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. It provides an opportunity to analyse and solve a wide range of geometrical problems in areas such as measurement, scaling, triangulation and navigation. It also provides opportunities to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series.

Mathematical Methods focuses on the development of the use of calculus and statistical analysis. The study of calculus in Mathematical Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Mathematical Methods develops the ability to describe and analyse phenomena involving uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. Specialist Mathematics is the only mathematics subject that has been designed to not be taken as a stand-alone subject.

Structure of Essential Mathematics

Essential Mathematics has four units each of which contains a number of topics. It is intended that the topics be taught in a context relevant to students' needs and interests. In Essential Mathematics, students use their knowledge and skills to investigate realistic problems of interest which involve the application of mathematical relationships and concepts.

Unit 1	Unit 2	Unit 3	Unit 4
Calculations, percentages and rates Measurement Algebra Graphs	Representing and comparing data Percentages Rates and ratios Time and motion	Measurement Scales, plans and models Graphs Data collection	Probability and relative frequencies Earth geometry and time zones Loans and compound interest

Units

Unit 1 provides students with the mathematical skills and understanding to solve problems relating to calculations, applications of measurement, the use of formulas to find an unknown quantity, and the interpretation of graphs. Teachers are encouraged to apply the content of all topics in contexts which are meaningful and of interest to their students. A variety of approaches could be used to achieve this. Two contexts which could be used in this unit are *Mathematics and foods* and *Earning and managing money*. However, these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Unit 2 provides students with the mathematical skills and understanding to solve problems related to representing and comparing data, percentages, rates and ratios, and time and motion. Teachers are encouraged to apply the content of all topics in contexts which are meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Mathematics and cars* and *Mathematics and independent living*. However these contexts may not be relevant for all students, and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Unit 3 provides students with the mathematical skills and understanding to solve problems related to measurement, scales, plans and models, drawing and interpreting graphs, and data collection. Teachers are encouraged to apply the content of all topics in contexts which are meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit to achieve this goal are *Mathematics and design* and *Mathematics and medicine*. However these contexts may not be relevant for all students and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Unit 4 provides students with the mathematical skills and understanding to solve problems related to probability, earth geometry and time zones, and loans and compound interest. Teachers are encouraged to apply the content of all topics in contexts which are meaningful and of interest to the students. A variety of approaches could be used to achieve this purpose. Two possible contexts which could be used in this unit are Mathematics of Finance and Mathematics of travelling. However these contexts may not be relevant for all students and teachers are encouraged to find a suitable context that will make the mathematical topics of this unit relevant for their particular student cohort.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: 'Concepts and Techniques' and 'Reasoning and Communication'. These two dimensions reflect students' understanding and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained *a priori* without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

- Given a specified context (as described in the curriculum content)
- With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
- Students perform a specified action (described through a verb)
- In relation to what is valued in the curriculum (specified as the object or subject)
- With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as 'analyse' and 'describe' have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

Links to Foundation to Year 10

For all content areas of Essential Mathematics, the proficiency strands of Understanding, Fluency, Problem solving and Reasoning from the F–10 curriculum are still very much applicable and should be inherent in students' learning of the subject. Each strand is essential, and all are mutually reinforcing. For all content areas, practice allows students to develop fluency in their skills. They will encounter opportunities for problem solving, such as finding the volume of a solid to enable the amount of liquid that is held in the container to be compared with what is written on the label, or finding the interest on an amount in order to be able to compare different types of loans. In Essential Mathematics, reasoning includes critically interpreting and analysing information represented through graphs, tables and other statistical representations to make informed decisions. The ability to transfer mathematical skills between contexts is a vital part of learning in this subject. For example, familiarity with the concept of a rate enables students to solve a wide range of practical problems, such as fuel consumption, travel times, interest payments, taxation, and population growth.

Representation of General capabilities

The seven general capabilities of *Literacy, Numeracy, Information and Communication Technology (ICT) capability, Critical and creative thinking, Personal and social capability, Ethical understanding*, and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains financial applications of Mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age. Students will also learn about the probability of certain events occurring and will therefore be well equipped to make informed decisions about gambling.

ICT in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical understanding in Mathematics

In the senior years students develop ethical behaviour in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical behaviour. Students develop increasingly advanced communication, research and presentation skills to express viewpoints.

Intercultural understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Representation of Cross-curriculum priorities

The Senior Secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples histories and cultures.

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. Through analysis of relevant data, students are provided with opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Each of the senior Mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Therefore, teachers are encouraged to select contexts for discussion connected with sustainability. Through analysis of data, students have the opportunity to research and discuss this global issue and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 1

Unit Description

This unit provides students with the mathematical skills and understanding to solve problems relating to calculations, applications of measurement, the use of formulas to find an unknown quantity, and the interpretation of graphs. Teachers are encouraged to apply the content of the four topics in this unit – 'Calculations, percentages and rates', 'Measurement', 'Algebra' and 'Graphs' – in contexts which are meaningful and of interest to their students. A variety of approaches can be used to achieve this purpose. Two possible contexts which may be used are *Mathematics and foods* and *Earning and managing money*. However, as these contexts may not be relevant to all students, teachers are encouraged to find suitable contexts relevant to their particular student cohort.

It is assumed that an extensive range of technological applications and techniques will be used in teaching this unit. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills.

Learning Outcomes

By the end of this unit students:

- understand the concepts and techniques in calculations, measurement, algebra and graphs
- apply reasoning skills and solve practical problems in calculations, measurement, algebra and graphs
- communicate their arguments and strategies when solving problems using appropriate mathematical language
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Calculations, percentages and rates

Calculations:

- solve practical problems requiring basic number operations (ACMEM001)
- apply arithmetic operations according to their correct order (ACMEM002)
- ascertain the reasonableness of answers to arithmetic calculations (ACMEM003)
- use leading-digit approximation to obtain estimates of calculations (ACMEM004)
- use a calculator for multi-step calculations (ACMEM005)
- check results of calculations for accuracy (ACMEM006)
- recognise the significance of place value after the decimal point (ACMEM007)
- evaluate decimal fractions to the required number of decimal places (ACMEM008)
- round up or round down numbers to the required number of decimal places (ACMEM009)
- apply approximation strategies for calculations. (ACMEM010)

Percentages:

- calculate a percentage of a given amount (ACMEM011)
- determine one amount expressed as a percentage of another (ACMEM012)
- apply percentage increases and decreases in situations; for example, mark-ups, discounts and GST. (ACMEM013)

Rates:

- identify common usage of rates; for example, km/h as a rate to describe speed, beats/minute as a rate to describe pulse (ACMEM014)
- convert units of rates occurring in practical situations to solve problems (ACMEM015)
- use rates to make comparisons; for example, using unit prices to compare best buys, comparing heart rates after exercise. (ACMEM016)

Topic 2: Measurement

Linear measure:

- use metric units of length, their abbreviations, conversions between them, and appropriate levels of accuracy and choice of units (ACMEM017)
- estimate lengths (ACMEM018)
- convert between metric units of length and other length units (ACMEM019)
- calculate perimeters of familiar shapes, including triangles, squares, rectangles, and composites of these. (ACMEM020)

Area measure:

- use metric units of area, their abbreviations, conversions between them, and appropriate choices of units (ACMEM021)
- estimate the areas of different shapes (ACMEM022)
- convert between metric units of area and other area units (ACMEM023)

- calculate areas of rectangles and triangles. (ACMEM024)

Mass:

- use metric units of mass, their abbreviations, conversions between them, and appropriate choices of units (ACMEM025)
- estimate the mass of different objects. (ACMEM026)

Volume and capacity:

- use metric units of volume, their abbreviations, conversions between them, and appropriate choices of units (ACMEM027)
- understand the relationship between volume and capacity (ACMEM028)
- estimate volume and capacity of various objects (ACMEM029)
- calculate the volume of objects, such as cubes and rectangular and triangular prisms. (ACMEM030)

Units of energy:

- use units of energy to describe consumption of electricity, such as kilowatt hours (ACMEM031)
- use units of energy used for foods, including calories (ACMEM032)
- use units of energy to describe the amount of energy in activity, such as kilojoules (ACMEM033)
- convert from one unit of energy to another. (ACMEM034)

Topic 3: Algebra

Single substitution:

- substitute numerical values into algebraic expressions; for example, substitute different values of *x* to evaluate the expressions $\frac{3x}{5}$, $5\left(2x-4\right)$. (ACMEM035)

General substitution:

- substitute given values for the other pronumerals in a mathematical formula to find the value of the subject of the formula. (ACMEM036)

Topic 4: Graphs

Reading and interpreting graphs:

- interpret information presented in graphs, such as conversion graphs, line graphs, step graphs, column graphs and picture graphs (ACMEM037)
- interpret information presented in two-way tables (ACMEM038)
- discuss and interpret graphs found in the media and in factual texts. (ACMEM039)

Drawing graphs:

- determine which type of graph is best used to display a dataset (ACMEM040)
- use spreadsheets to tabulate and graph data (ACMEM041)
- draw a line graph to represent any data that demonstrate a continuous change, such as hourly temperature. (ACMEM042)

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Unit 2

Unit Description

This unit provides students with the mathematical skills and understanding to solve problems related to representing and comparing data, percentages, rates and ratios, the mathematics of finance, and time and motion. Teachers are encouraged to apply the content of the four topics in this unit – ‘Representing and comparing data’, ‘Percentages’, ‘Rates and ratios’ and ‘Time and motion’ – in a context which is meaningful and of interest to their students. A variety of approaches can be used to achieve this purpose. Two possible contexts which may be used are *Mathematics and cars* and *Mathematics and independent living*. However, as these contexts may not be relevant to all students, teachers are encouraged to find suitable contexts relevant to their particular student cohort.

It is assumed that an extensive range of technological applications and techniques will be used in teaching this unit. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques used in representing and comparing data, percentages, rates and ratios, and time and motion
- apply reasoning skills and solve practical problems in representing and comparing data, percentages, rates and ratios, and time and motion
- communicate their arguments and strategies when solving mathematical and statistical problems using appropriate mathematical or statistical language
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Representing and comparing data

Classifying data:

- identify examples of categorical data (ACMEM043)
- identify examples of numerical data. (ACMEM044)

Data presentation and interpretation:

- display categorical data in tables and column graphs (ACMEM045)
- display numerical data as frequency distributions, dot plots, stem and leaf plots, and histograms (ACMEM046)
- recognise and identify outliers (ACMEM047)
- compare the suitability of different methods of data presentation in real-world contexts. (ACMEM048)

Summarising and interpreting data:

- identify the mode (ACMEM049)
- calculate measures of central tendency, the arithmetic mean and the median (ACMEM050)
- investigate the suitability of measures of central tendency in various real-world contexts (ACMEM051)
- investigate the effect of outliers on the mean and the median (ACMEM052)
- calculate and interpret quartiles, deciles and percentiles (ACMEM053)
- use informal ways of describing spread, such as spread out/dispersed, tightly packed, clusters, gaps, more/less dense regions, outliers (ACMEM054)
- calculate and interpret statistical measures of spread, such as the range, interquartile range and standard deviation (ACMEM055)
- investigate real-world examples from the media illustrating inappropriate uses, or misuses, of measures of central tendency and spread. (ACMEM056)

Comparing data sets:

- compare back-to-back stem plots for different data-sets (ACMEM057)
- complete a five number summary for different datasets (ACMEM058)
- construct box plots using a five number summary (ACMEM059)
- compare the characteristics of the shape of histograms using symmetry, skewness and bimodality. (ACMEM060)

Topic 2: Percentages

Percentage calculations:

- review calculating a percentage of a given amount (ACMEM061)
- review one amount expressed as a percentage of another. (ACMEM062)

Applications of percentages:

- determine the overall change in a quantity following repeated percentage changes; for example, an increase of 10% followed by a decrease of 10% (ACMEM063)

- calculate simple interest for different rates and periods. (ACMEM064)

Topic 3: Rates and ratios

Ratios:

- demonstrate an understanding of the elementary ideas and notation of ratio (ACMEM065)
- understand the relationship between fractions and ratio (ACMEM066)
- express a ratio in simplest form (ACMEM067)
- find the ratio of two quantities (ACMEM068)
- divide a quantity in a given ratio (ACMEM069)
- use ratio to describe simple scales. (ACMEM070)

Rates:

- review identifying common usage of rates such as km/h (ACMEM071)
- convert between units for rates; for example, km/h to m/s, mL/min to L/h (ACMEM072)
- complete calculations with rates, including solving problems involving direct proportion in terms of rate. (ACMEM073)
- use rates to make comparisons (ACMEM074)
- use rates to determine costs; for example, calculating the cost of a tradesman using rates per hour, call-out fees. (ACMEM075)

Topic 4: Time and motion

Time:

- use units of time, conversions between units, fractional, digital and decimal representations (ACMEM076)
- represent time using 12-hour and 24-hour clocks (ACMEM077)
- calculate time intervals, such as time between, time ahead, time behind (ACMEM078)
- interpret timetables, such as bus, train and ferry timetables (ACMEM079)
- use several timetables and electronic technologies to plan the most time-efficient routes (ACMEM080)
- interpret complex timetables, such as tide charts, sunrise charts and moon phases (ACMEM081)
- compare the time taken to travel a specific distance with various modes of transport (ACMEM082)

Distance:

- use scales to find distances, such as on maps; for example, road maps, street maps, bushwalking maps, online maps and cadastral maps (ACMEM083)
- optimise distances through trial-and-error and systematic methods; for example, shortest path, routes to visit all towns, and routes to use all roads. (ACMEM084)

Speed:

- identify the appropriate units for different activities, such as walking, running, swimming and flying (ACMEM085)
- calculate speed, distance or time using the formula *speed = distance/time* (ACMEM086)
- calculate the time or costs for a journey from distances estimated from maps (ACMEM087)

- interpret distance-versus-time graphs (ACMEM088)
- calculate and interpret average speed; for example, a 4-hour trip covering 250 km. (ACMEM089)

Essential Mathematics

Units 1 and 2 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of measurement, financial mathematics and statistics in routine and non-routine problems in a variety of contexts • selects and applies techniques in measurement, financial mathematics and statistics to solve routine and non-routine problems in a variety of contexts • uses digital technologies effectively to display and organise mathematical and statistical information to solve routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of measurement, financial mathematics and statistics in routine and non-routine problems • selects and applies techniques in measurement, financial mathematics and statistics to solve routine and non-routine problems • uses digital technologies appropriately to display and organise mathematical and statistical information to solve routine and non-routine problems	• demonstrates knowledge of concepts of measurement, financial mathematics and statistics in routine problems • selects and applies techniques in measurement, financial mathematics and statistics to solve routine problems • uses digital technologies to display and organise mathematical and statistical information to solve routine problems	• demonstrates familiarity with concepts of measurement, financial mathematics and statistics • uses simple techniques in measurement, financial mathematics and statistics • uses digital technologies to display and organise simple mathematical and statistical information	• demonstrates limited familiarity with concepts of measurement, financial mathematics or statistics • uses simple techniques in a structures context • uses digital technologies for arithmetic calculations

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates clear and reasoned observations and judgments using appropriate mathematical and statistical language • interprets solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of results and solutions to routine and non-routine problems in a variety of contexts	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear observations and judgments using appropriate mathematical and statistical language • interprets solutions to routine and non-routine problems • explains the reasonableness of results and solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates observations and judgments using appropriate mathematical and statistical language • interprets solutions to routine problems • describes the reasonableness of results and solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical and symbolic form • describes observations using mathematical and statistical language • describes solutions to routine problems • describes the appropriateness of the results of calculations	• represents simple mathematical and statistical information in a structured context • describes simple observations • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations

Unit 3

Unit Description

This unit provides students with the mathematical skills and understanding to solve problems related to measurement, scales, plans and models, drawing and interpreting graphs, and data collection. Teachers are encouraged to apply the content of the four topics in this unit – 'Measurement', 'Scales, plans and models', 'Graphs' and 'Data collection' – in a context which is meaningful and of interest to the students. A variety of approaches can be used to achieve this purpose. Two possible contexts which may be used in this unit are *Mathematics and design* and *Mathematics and medicine*. However, as these contexts may not be relevant to all students, teachers are encouraged to find suitable contexts relevant to their particular student cohort.

It is assumed that an extensive range of technological applications and techniques will be used in teaching this unit. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques used in measurement, scales, plans and models, graphs, and data collection
- apply reasoning skills and solve practical problems in measurement, scales, plans and models, graphs, and data collection
- communicate their arguments and strategies when solving mathematical and statistical problems using appropriate mathematical or statistical language
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Measurement

Linear measure:

- review metric units of length, their abbreviations, conversions between them, estimation of lengths, and appropriate choices of units (ACMEM090)
- calculate perimeters of familiar shapes, including triangles, squares, rectangles, polygons, circles, arc lengths, and composites of these. (ACMEM091)

Area measure:

- review metric units of area, their abbreviations, and conversions between them (ACMEM092)
- use formulas to calculate areas of regular shapes, including triangles, squares, rectangles, parallelograms, trapeziums, circles and sectors (ACMEM093)
- find the area of irregular figures by decomposition into regular shapes (ACMEM094)
- find the surface area of familiar solids, including cubes, rectangular and triangular prisms, spheres and cylinders (ACMEM095)
- find the surface area of pyramids, such as rectangular- and triangular-based pyramids (ACMEM096)
- use addition of the area of the faces of solids to find the surface area of irregular solids. (ACMEM097)

Mass:

- review metric units of mass (and weight), their abbreviations, conversions between them, and appropriate choices of units (ACMEM098)
- recognise the need for milligrams (ACMEM099)
- convert between grams and milligrams. (ACMEM100)

Volume and capacity:

- review metric units of volume, their abbreviations, conversions between them, and appropriate choices of units (ACMEM101)
- recognise relations between volume and capacity, recognising that $1\text{cm}^3 = 1\text{mL}$ and $1\text{m}^3 = 1\text{kL}$ (ACMEM102)
- use formulas to find the volume and capacity of regular objects such as cubes, rectangular and triangular prisms and cylinders (ACMEM103)
- use formulas to find the volume of pyramids and spheres. (ACMEM104)

Topic 2: Scales, plans and models

Geometry:

- recognise the properties of common two-dimensional geometric shapes and three-dimensional solids (ACMEM105)
- interpret different forms of two-dimensional representations of three-dimensional objects, including nets and perspective diagrams (ACMEM106)
- use symbols and conventions for the representation of geometric information; for example, point, line, ray, angle, diagonal,

edge, curve, face and vertex. (ACMEM107)

Interpret scale drawings:

- interpret commonly used symbols and abbreviations in scale drawings (ACMEM108)
- find actual measurements from scale drawings, such as lengths, perimeters and areas (ACMEM109)
- estimate and compare quantities, materials and costs using actual measurements from scale drawings; for example, using measurements for packaging, clothes, painting, bricklaying and landscaping. (ACMEM110)

Creating scale drawings:

- understand and apply drawing conventions of scale drawings, such as scales in ratio, clear indications of dimensions, and clear labelling (ACMEM111)
- construct scale drawings by hand and by using software packages. (ACMEM112)

Three dimensional objects:

- interpret plans and elevation views of models (ACMEM113)
- sketch elevation views of different models (ACMEM114)
- interpret diagrams of three-dimensional objects. (ACMEM115)

Right-angled triangles:

- apply Pythagoras' theorem to solve problems (ACMEM116)
- apply the tangent ratio to find unknown angles and sides in right-angled triangles (ACMEM117)
- work with the concepts of angle of elevation and angle of depression (ACMEM118)
- apply the cosine and sine ratios to find unknown angles and sides in right-angled triangles (ACMEM119)
- solve problems involving bearings. (ACMEM120)

Topic 3: Graphs

Cartesian plane:

- demonstrate familiarity with Cartesian coordinates in two dimensions by plotting points on the Cartesian plane (ACMEM121)
- generate tables of values for linear functions, including for negative values of x (ACMEM122)
- graph linear functions for all values of x with pencil and paper and with graphing software. (ACMEM123)

Using graphs:

- interpret and use graphs in practical situations, including travel graphs and conversion graphs (ACMEM124)
- draw graphs from given data to represent practical situations (ACMEM125)
- interpret the point of intersection and other important features of given graphs of two linear functions drawn from practical contexts; for example, the 'break-even' point. (ACMEM126)

Topic 4: Data collection

Census:

- investigate the procedure for conducting a census (ACMEM127)

- investigate the advantages and disadvantages of conducting a census. (ACMEM128)

Surveys:

- understand the purpose of sampling to provide an estimate of population values when a census is not used (ACMEM129)
- investigate the different kinds of samples; for example, systematic samples, self-selected samples, simple random samples (ACMEM130)
- investigate the advantages and disadvantages of these kinds of samples; for example, comparing simple random samples with self-selected samples. (ACMEM131)

Simple survey procedure:

- identify the target population to be surveyed (ACMEM132)
- investigate questionnaire design principles; for example, simple language, unambiguous questions, consideration of number of choices, issues of privacy and ethics, and freedom from bias. (ACMEM133)

Sources of bias:

- describe the faults in the collection of data process (ACMEM134)
- describe sources of error in surveys; for example, sampling error and measurement error (ACMEM135)
- investigate the possible misrepresentation of the results of a survey due to misunderstanding the procedure, or misunderstanding the reliability of generalising the survey findings to the entire population (ACMEM136)
- investigate errors and misrepresentation in surveys, including examples of media misrepresentations of surveys. (ACMEM137)

Bivariate scatterplots:

- describe the patterns and features of bivariate data (ACMEM138)
- describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak). (ACMEM139)

Line of best fit:

- identify the dependent and independent variable (ACMEM140)
- find the line of best fit by eye (ACMEM141)
- use technology to find the line of best fit (ACMEM142)
- interpret relationships in terms of the variables (ACMEM143)
- use technology to find the correlation coefficient (an indicator of the strength of linear association) (ACMEM144)
- use the line of best fit to make predictions, both by interpolation and extrapolation (ACMEM145)
- recognise the dangers of extrapolation (ACMEM146)
- distinguish between causality and correlation through examples. (ACMEM147)

Unit 4

Unit Description

This unit provides students with the mathematical skills and understanding to solve problems related to probability, Earth geometry and time zones, and loans and compound interest. Teachers are encouraged to apply the content of the three topics in this unit – 'Probability and relative frequencies', 'Earth geometry and time zones' and 'Loans and compound interest' – in a context which is meaningful and of interest to the students. A variety of approaches can be used to achieve this purpose. Two possible contexts which may be used in this unit are Mathematics of finance and Mathematics of travelling. However, as these contexts may not be relevant to all students, teachers are encouraged to find suitable contexts relevant to their particular student cohort.

It is assumed that an extensive range of technological applications and techniques will be used in teaching this unit. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques used in probability and relative frequencies, earth geometry and time zones, loans and compound interest
- apply reasoning skills and solve practical problems in probability and relative frequencies, earth geometry and time zones, loans and compound interest
- communicate their arguments and strategies when solving mathematical problems using appropriate mathematical or statistical language
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Probability and relative frequencies

Probability expressions:

- interpret commonly used probability statements, including ‘possible’, ‘probable’, ‘likely’, ‘certain’ (ACMEM148)
- describe ways of expressing probabilities formally using fractions, decimals, ratios, and percentages. (ACMEM149)

Simulations:

- perform simulations of experiments using technology (ACMEM150)
- recognise that the repetition of chance events is likely to produce different results (ACMEM151)
- identify relative frequency as probability (ACMEM152)
- identify factors that could complicate the simulation of real-world events. (ACMEM153)

Simple probabilities:

- construct a sample space for an experiment (ACMEM154)
- use a sample space to determine the probability of outcomes for an experiment (ACMEM155)
- use arrays or tree diagrams to determine the outcomes and the probabilities for experiments. (ACMEM156)

Probability applications

- determine the probabilities associated with simple games (ACMEM157)
- determine the probabilities of occurrence of simple traffic-light problems. (ACMEM158)

Topic 2: Earth geometry and time zones

Location:

- locate positions on Earth’s surface given latitude and longitude using GPS, a globe, an atlas, and digital technologies (ACMEM159)
- find distances between two places on Earth on the same longitude (ACMEM160)
- find distances between two places on Earth using appropriate technology. (ACMEM161)

Time:

- understand the link between longitude and time (ACMEM162)
- solve problems involving time zones in Australia and in neighbouring nations, making any necessary allowances for daylight saving (ACMEM163)
- solve problems involving Greenwich Mean Time and the International Date Line (ACMEM164)
- find time differences between two places on Earth (ACMEM165)
- solve problems associated with time zones; for example, internet and phone usage (ACMEM166)
- solve problems relating to travelling east and west, incorporating time zone changes. (ACMEM167)

Topic 3: Loans and compound interest

Compound interest:

- review the principles of simple interest (ACMEM168)
- understand the concept of compound interest as a recurrence relation (ACMEM169)
- consider similar problems involving compounding; for example, population growth (ACMEM170)
- use technology to calculate the future value of a compound interest loan or investment and the total interest paid or earned (ACMEM171)
- use technology to compare, numerically and graphically, the growth of simple interest and compound interest loans and investments (ACMEM172)
- use technology to investigate the effect of the interest rate and the number of compounding periods on the future value of a loan or investment. (ACMEM173)

Reducing balance loans (compound interest loans with periodic repayments):

- use technology and a recurrence relation to model a reducing balance loan (ACMEM174)
- investigate the effect of the interest rate and repayment amount on the time taken to repay a loan. (ACMEM175)

Units 3 and 4 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of measurement, scales, graphs and statistics in routine and non-routine problems in a variety of contexts • selects and applies techniques in measurement, scales, graphs and statistics to solve routine and non-routine problems in a variety of contexts • uses digital technologies effectively to display and organise mathematical and statistical information to solve routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of measurement, scales, graphs and statistics in routine and non-routine problems • selects and applies techniques in measurement, scales, graphs and statistics to solve routine and non-routine problems • uses digital technologies appropriately to display and organise mathematical and statistical information to solve routine and non-routine problems	• demonstrates knowledge of concepts of measurement, scales, graphs and statistics in routine problems • selects and applies techniques in measurement, scales, graphs and statistics to solve routine problems • uses digital technologies to display and organise mathematical and statistical information to solve routine problems	• demonstrates familiarity with concepts of measurement, scales, graphs and statistics • uses simple techniques in measurement, scales, graphs and statistics • uses digital technologies to display and organise simple mathematical and statistical information	• demonstrates limited familiarity with concepts of measurement, scales, graphs and statistics • uses simple techniques in a structured context • uses digital technologies for arithmetic calculations

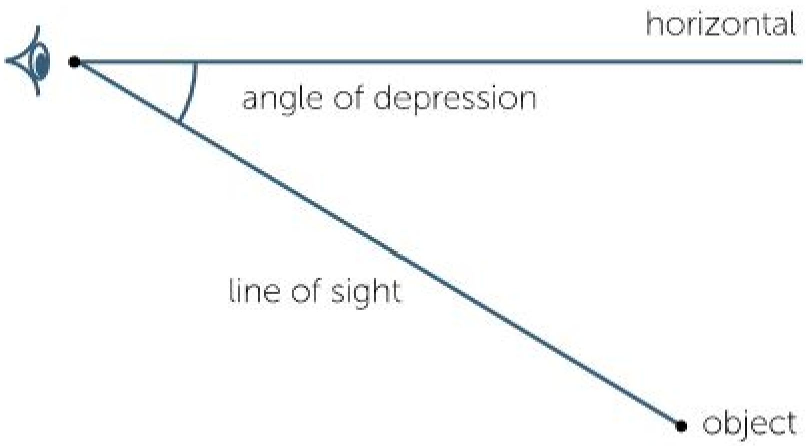
Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates clear and reasoned observations and judgments using appropriate mathematical and statistical language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of results and solutions to routine and non-routine problems in a variety of contexts	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear observations and judgments using appropriate mathematical and statistical language • interprets the solutions to routine and non-routine problems • explains the reasonableness of results and solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates observations and judgments using appropriate mathematical and statistical language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical and symbolic form • describes observations using mathematical and statistical language • describes the solutions to routine problems • describes the appropriateness of the results of calculations	• represents simple mathematical and statistical information in a structured context • describes simple observations • identifies the solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations

Essential Mathematics Glossary

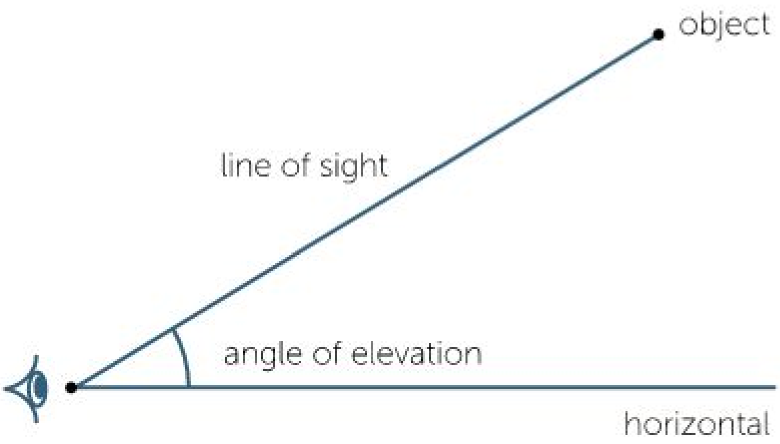
Angle of depression

When an observer looks at an object that is lower than 'the eye of' the observer', the angle between the line of sight and the horizontal is called the angle of depression.



Angle of elevation

When an observer looks at an object that is higher than 'the eye of' the observer', the angle between the line of sight and the horizontal is called the angle of elevation.



Average speed

Average speed is the total distance travelled divided by the total time taken.

Back-to back stem plots

A back-to-back stem-and-leaf plot is a method for comparing two data distributions by attaching two sets of 'leaves' to the same 'stem' in a stem-and-leaf plot.

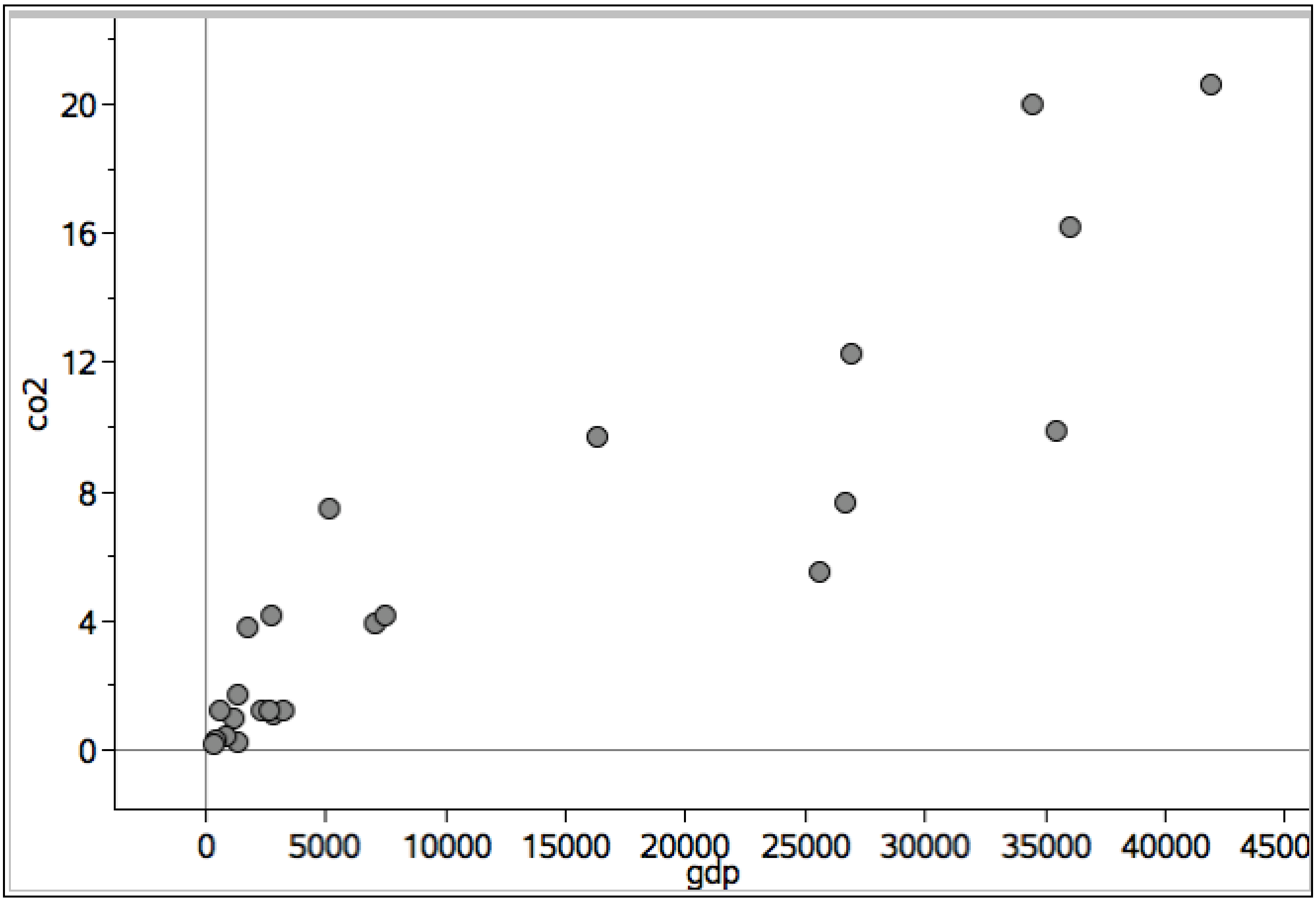
For example, the stem-and-leaf plot below displays the distribution of pulse rates of 19 students before and after gentle exercise.

before	after	pulse rate
9 8 8 8	6	
8 6 6 4 1 1	7	
8 8 6 2	8	6 7 8 8
6 0	9	0 2 2 4 5 8 9 9
4	10	0 4 4
0	11	8
	12	4 4
	13	

Bivariate data scatter plot

A two-dimensional data plot using Cartesian co-ordinates to display the values of two variables in a bivariate data set.

For example the scatterplot below displays the CO2 emissions in tonnes per person (*co2*) plotted against Gross Domestic Product per person in $US (*gdp*) for a sample of 24 countries in 2004. In constructing this scatterplot, gdp has been used as the explanatory variable.



Categorical data

Data associated with a categorical variable is called categorical data.

Categorical variable

A categorical variable is a variable whose values are categories.

Examples include blood group (A, B, AB or O) or house construction type (brick, concrete, timber, steel, other).

Categories may have numerical labels, eg. the numbers worn by player in a sporting team, but these labels have no numerical significance, they merely serve as labels.

Census

A population is the complete set of individuals, objects, places, etc, that we want information about.

A census is an attempt to collect information about the whole population.

Clarks' formula

Dosage for children (general formula) = (weight in kg x adult dosage) /70

Compound interest

The interest earned by investing a sum of money (the principal) is compound interest if each successive interest payment is added to the principal for the purpose of calculating the next interest payment.

For example, if the principal *P* earns compound interest at the rate of *i* % per period, then after *n* periods the total amount accrued is $P\left(1+\ \frac{i}{100}\right)^{n}$

Correlation

Correlation is a measure of the strength of the linear relationship between two variables.

Correlation coefficient

The correlation coefficient (*r*) is a measure of the strength of the liner relationship between a pair of variables. The formula for calculating *r* is given below.

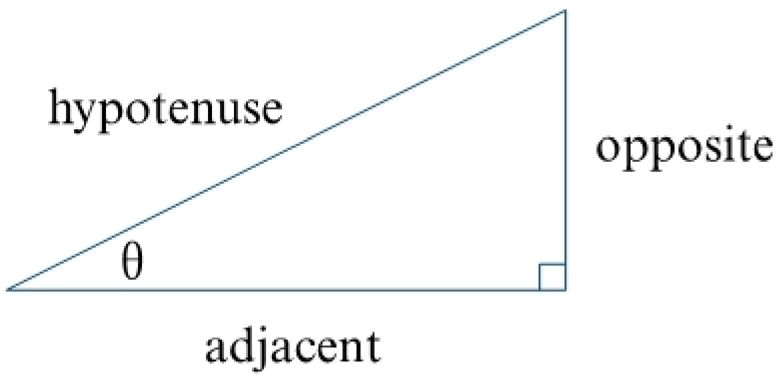
For variables *x* and *y*, and computed for *n* cases, the formula for *r* is:

$$r=\ \frac{1}{n-1}\sum\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right)$$

Cosine ratio

In any right-angled triangle,

$$\cos\theta=\frac{\text{adjacent}}{\text{hypotenuse}}\quad \text{where}\quad 0^\circ<\theta<90^\circ$$



Extrapolation

In the context of fitting a linear relationship between two variables, extrapolation occurs when the fitted model is used to make predictions using values of the explanatory variable that are outside the range of the original data. Extrapolation is a dangerous process as it can sometimes lead to quite erroneous predictions.

Five-number summary

A five-number summary is a method of summarising a set of data using the minimum value, the lower or first-quartile (Q_1), the median, the upper or third-quartile (Q_3) and the maximum value. Forms the basis for a boxplot.

Frieds’ formula

Dosage for children 1-2 years = (age (in months) x adult dosage) /150

GST

The GST (Goods and Services Tax) is a broad sales tax of 10% on most goods and services transactions in Australia.

Interquartile range

The interquartile range (IQR) is a measure of the spread within a numerical data set. It is equal to the upper quartile (Q_3) minus the lower quartiles (Q_1); that is, $IQR = Q_3 - Q_1$

The IQR is the width of an interval that contains the middle 50% (approximately) of the data values. To be exactly 50%, the sample size must be a multiple of four.

kWh (kilowatt hour)

The kilowatt hour, or kilowatt-hour, is a unit of energy equal to 1000 watt hours or 3.6 megajoules The kilowatt hour is most commonly known as a billing unit for energy delivered to consumers by electric utilities.

Mean

The arithmetic mean of a list of numbers is the sum of the data values divided by the number of values in the list.

In everyday language, the arithmetic mean is commonly called the average.

For example, for the following list of five numbers 2, 3, 3, 6, 8 the mean equals

$$\frac{2+3+3+6+8}{5} = \frac{22}{5} = 4.4$$

In more general language, the mean of n observations $x_1, x_2, \ldots, x_n$ is

$$\overline{x} = \frac{\sum x_i}{n}$$

Median

The median is the value in a set of ordered set of data values that divides the data into two parts of equal size. When there are an odd number of data values, the median is the middle value. When there is an even number of data values, the median is the average of the two central values.

MJ (Megajoule)

A joule is the SI unit of work. The megajoule (MJ) is equal to one million joules

Mode

The mode is the most frequently occurring value is a data set.

Outlier

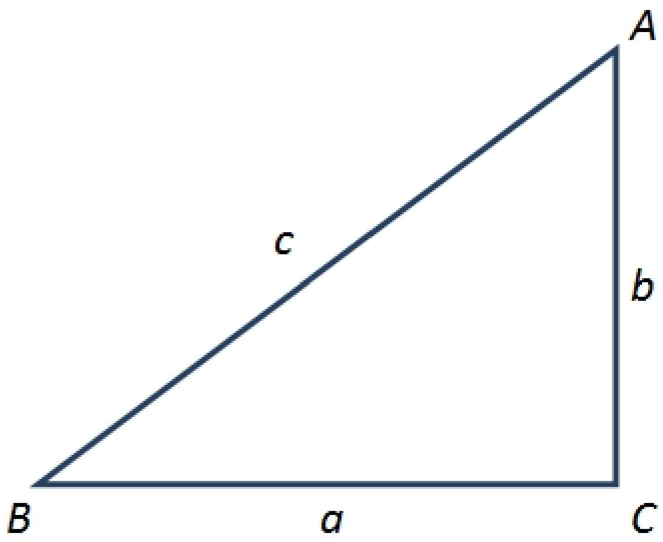
An outlier in a set of data is an observation that appears to be inconsistent with the remainder of that set of data. An outlier is a surprising observation.

Pythagoras' theorem

For a right-angled triangle

The square of the hypotenuse of a right-angled triangle equals the sum of the squares of the lengths of the other two sides.

In symbols, $c^2 = a^2 + b^2$.



Range

The range is the difference between the largest and smallest observations in a data set.

The range can be used as a measure of spread in a data set, but it is extremely sensitive to the presence of outliers and should only be used with care.

Reaction time

The time a person takes to react to a situation (pressing the brake) requiring them to stop

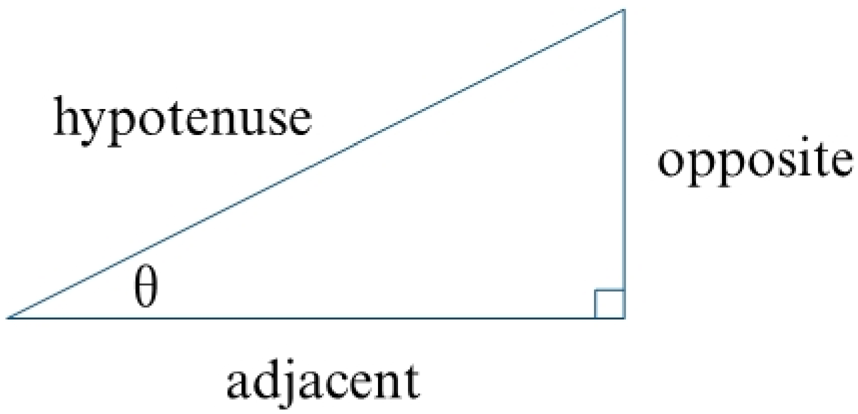
Simple interest

Simple interest is the interest accumulated when the interest payment in each period is a fixed fraction of the principal. For example, if the principle *P* earns simple interest at the rate of *i* % per period, then after *n* periods the accumulated simple interest is $\frac{Pni}{100}$

Sine ratio

In any right-angled triangle,

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \text{ where } \quad 0^\circ < \theta < 90^\circ$$



Stopping distances

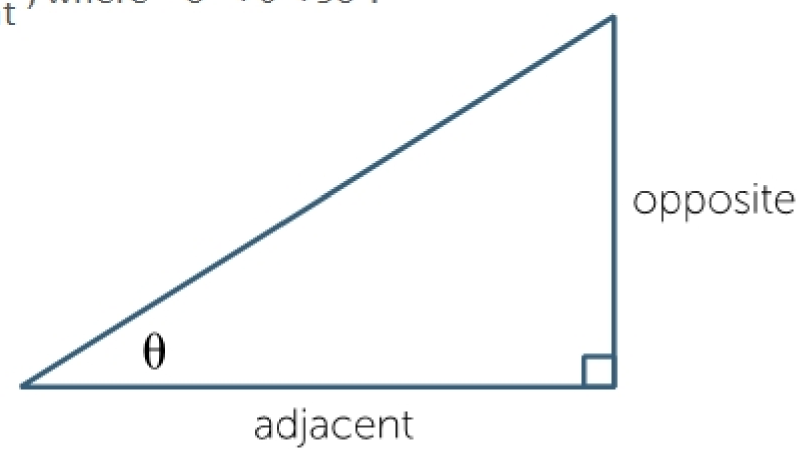
The distance a car travels after the driver has applied the brake given speed of the vehicle and/or conditions of the road which can be found using formula or tables.

Stopping distance = braking distance + reaction time(secs) Xspeed

Tangent ratio

In any right-angled triangle*,*

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}, \text{ where } \quad 0^\circ < \theta < 90^\circ.$$



The converse

If $c^2 = a^2 + b^2$ in a triangle *ABC*, then ∠C is a right angle.

Young’s formula

Dosage for Children 1-12 years = (weight in kg x adult dosage)/ (age of child (in years) + 12)

The Australian Curriculum
General Mathematics

AUSTRALIAN CURRICULUM, ASSESSMENT AND REPORTING AUTHORITY

General Mathematics

Rationale and Aims

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe many aspects of the world in the twenty-first century. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve practical problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

General Mathematics is designed for those students who want to extend their mathematical skills beyond Year 10 level but whose future studies or employment pathways do not require knowledge of calculus. The subject is designed for students who have a wide range of educational and employment aspirations, including continuing their studies at university or TAFE.

The proficiency strands of the F-10 curriculum – Understanding, Fluency, Problem solving and Reasoning – are still relevant and are inherent in all aspects of this subject. Each of these proficiencies is essential, and all are mutually reinforcing. Fluency, for example, might include learning to perform routine calculations efficiently and accurately, or being able to recognise quickly from a problem description the appropriate mathematical process or model to apply. Understanding, furthermore, that a single mathematical process can be used in seemingly different situations, helps students to see the connections between different areas of study and encourages the transfer of learning. This is an important part of learning the art of mathematical problem solving. In performing such analyses, reasoning is required at each decision-making step and in drawing appropriate conclusions. Presenting the analysis in a logical and clear manner to explain the reasoning used is also an integral part of the learning process.

Throughout the subject there is also an emphasis on the use and application of digital technologies.

Aims

General Mathematics aims to develop students':

- understanding of concepts and techniques drawn from the topic areas of number and algebra, geometry and trigonometry, graphs and networks, and statistics
- ability to solve applied problems using concepts and techniques drawn from the topic areas of number and algebra, geometry and trigonometry, graphs and networks, and statistics
- reasoning and interpretive skills in mathematical and statistical contexts
- capacity to communicate the results of a mathematical or statistical problem-solving activity in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Organisation

Overview of senior secondary Australian Curriculum

ACARA has developed senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications. The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

Each senior secondary Australian Curriculum specifies content and achievement standards for a senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) demonstrated by students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a school year of senior secondary studies (approximately 50–60 hours duration including assessment). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, over a year), or as four units over two years. State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into courses. They will also provide any advice on entry and exit points, in line with their curriculum, assessment and certification requirements.

States and territories, through their respective curriculum, assessment and certification authorities, will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and board. They will determine the assessment and certification specifications for their courses that use the Australian Curriculum content and achievement standards, as well as any additional information, guidelines and rules to satisfy local requirements.

This curriculum should not, therefore, be read as proposed courses of study. Rather, it presents content and achievement standards for integration into state and territory courses.

Senior secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics, with each subject organised into four units. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. It also provides the opportunity for students to prepare for post-school options of employment or further training.

General Mathematics focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve analysing univariate and bivariate data, including time series data.

Mathematical Methods focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends students' understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. It is the only mathematics subject that cannot be taken as a stand-alone subject.

Structure of General Mathematics

General Mathematics is organised into four units. The topics in each unit broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic, geometric and statistical thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction.

Unit 1	Unit 2	Unit 3	Unit 4
Consumer arithmetic Algebra and matrices Shape and measurement	Univariate data analysis and the statistical investigation process Applications of trigonometry Linear equations and their graphs	Bivariate data analysis Growth and decay in sequences Graphs and networks	Time series analysis Loans, investments and annuities Networks and decision mathematics

Units

Unit 1 has three topics: 'Consumer arithmetic', 'Algebra and matrices', and 'Shape and measurement'. 'Consumer arithmetic' reviews the concepts of rate and percentage change in the context of earning and managing money, and provides fertile ground for the use of spreadsheets. 'Algebra and matrices' continues the F-10 study of algebra and introduces the new topic of matrices. 'Shape and measurement' extends the knowledge and skills students developed in the F-10 curriculum with the concept of similarity and associated calculations involving simple and compound geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Unit 2 has three topics: 'Univariate data analysis and the statistical investigation process', 'Linear equations and their graphs', and 'Applications of trigonometry'. 'Univariate data analysis and the statistical investigation process' develops students' ability to organise and summarise univariate data in the context of conducting a statistical investigation. . 'Applications of trigonometry' extends students' knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression, and bearings in navigation 'Linear equations and their graphs' uses linear equations and straight-line graphs, as well as linear-piecewise and step graphs, to model and analyse practical situations

Unit 3 has three topics: ‘Bivariate data analysis’, ‘Growth and decay in sequences’, and ‘Graphs and networks’. ‘Bivariate data analysis’ introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including using the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process. ‘Growth and decay in sequences’ employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4. ‘Graphs and networks’ introduces students to the language of graphs and the way in which graphs, represented as a collection of points and interconnecting lines, can be used to analyse everyday situations such as a rail or social network.

Unit 4 has three topics: ‘Time series analysis’, ‘Loans, investments and annuities’, and ‘Networks and decision mathematics’. ‘Time series analysis’ continues students’ study of statistics by introducing them to the concepts and techniques of time series analysis. The content is to be taught within the framework of the statistical investigation process. ‘Loans and investments’ aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments. ‘Networks and decision mathematics’ uses networks to model and aid decision making in practical situations.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained a priori without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

- Given a specified context (as described in the curriculum content)
- With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
- Students perform a specified action (described through a verb)
- In relation to what is valued in the curriculum (specified as the object or subject)
- With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as ‘analyse’ and ‘describe’ have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills in these subjects.

Links to Foundation to Year 10

The General Mathematics subject provides students with a breadth of mathematical and statistical experience that encompasses and builds on all three strands of the F-10 curriculum.

Representation of General capabilities

The seven general capabilities of *Literacy, Numeracy, Information and Communication Technology (ICT) capability, Critical and creative thinking, Personal and social capability, Ethical understanding*, and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in mathematics

In the senior years, literacy skills and strategies enable students to express, interpret and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their abilities to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in mathematics

The students who undertake this subject will develop their numeracy skills at a more sophisticated level than in Foundation to Year 10. This subject contains financial applications of mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age.

ICT in mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, using data, addressing problems, and operating systems in authentic situations.

Critical and creative thinking in mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions do not match, it is due to a flaw in the theory or in the method of applying the theory to make predictions, or both. They revise, or reapply, their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and in making accurate predictions.

Personal and social capability in mathematics

In the senior years students develop personal and social competence in mathematics by setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision making.

The elements of personal and social competence relevant to mathematics mainly include the application of mathematical skills for decision making, life-long learning, citizenship and self-management. As part of their mathematical explorations and investigations, students work collaboratively in teams, as well as independently.

Ethical understanding in mathematics

In the senior years students develop ethical understanding in mathematics through decision making connected with ethical dilemmas that arise when engaged in mathematical calculation, the dissemination of results, and the social responsibility associated with teamwork and attribution of input.

The areas relevant to mathematics include issues associated with ethical decision making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and the examined ethical behaviour. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding in mathematics

Students understand mathematics as a socially constructed body of knowledge that uses universal symbols but has its origins in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number, and that diverse cultural spatial abilities and understandings are shaped by a person’s environment and language.

Representation of Cross-curriculum priorities

The senior secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander peoples and their central place in contemporary Australian society and culture. Through the study of mathematics within relevant contexts, students are given opportunities to develop their understanding and appreciation of the diversity of cultures and histories of Aboriginal and Torres Strait Islander peoples and their contribution to Australian society.

There are strong social, cultural and economic reasons for Australian students to engage with Asia and with the contribution of Asian Australians to our society and heritage. It is through the study of mathematics in an Asian context that a creative and forward-looking Australia can truly engage with our place in the region. By analysing relevant data, students have opportunities to develop an understanding of the diversity of Asia’s peoples, environments, and traditional and contemporary cultures.

Each of the senior mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world views.

Unit 1

Unit Description

This unit has three topics: ‘Consumer arithmetic’, ‘Algebra and matrices’, and ‘Shape and measurement’.

‘Consumer arithmetic’ reviews the concepts of rate and percentage change in the context of earning and managing money, and provides a fertile ground for the use of spreadsheets.

‘Algebra and matrices’ continues the F-10 study of algebra and introduces the new topic of matrices.

‘Shape and measurement’ builds on and extends the knowledge and skills students developed in the F-10 curriculum with the concept of similarity and associated calculations involving simple and compound geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Classroom access to the technology necessary to support the computational aspects of the topics in this unit is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques introduced in consumer arithmetic, algebra and matrices, and shape and measurement
- apply reasoning skills and solve practical problems arising in consumer arithmetic, algebra and matrices, and shape and measurement
- communicate their arguments and strategies, when solving problems, using appropriate mathematical language
- interpret mathematical information, and ascertain the reasonableness of their solutions to problems
- choose and use technology appropriately and efficiently.

Content Descriptions

Topic 1: Consumer arithmetic

Applications of rates and percentages:

- review rates and percentages (ACMGM001)
- calculate weekly or monthly wage from an annual salary, wages from an hourly rate including situations involving overtime and other allowances and earnings based on commission or piecework (ACMGM002)
- calculate payments based on government allowances and pensions (ACMGM003)
- prepare a personal budget for a given income taking into account fixed and discretionary spending (ACMGM004)
- compare prices and values using the unit cost method (ACMGM005)
- apply percentage increase or decrease in various contexts; for example, determining the impact of inflation on costs and wages over time, calculating percentage mark-ups and discounts, calculating GST, calculating profit or loss in absolute and percentage terms, and calculating simple and compound interest (ACMGM006)
- use currency exchange rates to determine the cost in Australian dollars of purchasing a given amount of a foreign currency, such as US$1500, or the value of a given amount of foreign currency when converted to Australian dollars, such as the value of €2050 in Australian dollars (ACMGM007)
- calculate the dividend paid on a portfolio of shares, given the percentage dividend or dividend paid per share, for each share; and compare share values by calculating a price-to-earnings ratio. (ACMGM008)

Use of spreadsheets:

- use a spreadsheet to display examples of the above computations when multiple or repeated computations are required; for example, preparing a wage-sheet displaying the weekly earnings of workers in a fast food store where hours of employment and hourly rates of pay may differ, preparing a budget, or investigating the potential cost of owning and operating a car over a year. (ACMGM009)

Topic 2: Algebra and matrices

Linear and non-linear expressions:

- substitute numerical values into linear algebraic and simple non-linear algebraic expressions, and evaluate (ACMGM010)
- find the value of the subject of the formula, given the values of the other pronumerals in the formula (ACMGM011)
- use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities; for example, a table displaying the body mass index (BMI) of people of different weights and heights. (ACMGM012)

Matrices and matrix arithmetic:

- use matrices for storing and displaying information that can be presented in rows and columns; for example, databases, links in social or road networks (ACMGM013)
- recognise different types of matrices (row, column, square, zero, identity) and determine their size (ACMGM014)
- perform matrix addition, subtraction, multiplication by a scalar, and matrix multiplication, including determining the power of a matrix using technology with matrix arithmetic capabilities when appropriate (ACMGM015)
- use matrices, including matrix products and powers of matrices, to model and solve problems; for example, costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can

communicate with each other via a third person. (ACMGM016)

Topic 3: Shape and measurement

Pythagoras' Theorem:

- review Pythagoras' Theorem and use it to solve practical problems in two dimensions and for simple applications in three dimensions. (ACMGM017)

Mensuration:

- solve practical problems requiring the calculation of perimeters and areas of circles, sectors of circles, triangles, rectangles, parallelograms and composites (ACMGM018)
- calculate the volumes of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the volume of water contained in a swimming pool (ACMGM019)
- calculate the surface areas of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the surface area of a cylindrical food container. (ACMGM020)

Similar figures and scale factors:

- review the conditions for similarity of two-dimensional figures including similar triangles (ACMGM021)
- use the scale factor for two similar figures to solve linear scaling problems (ACMGM022)
- obtain measurements from scale drawings, such as maps or building plans, to solve problems (ACMGM023)
- obtain a scale factor and use it to solve scaling problems involving the calculation of the areas of similar figures (ACMGM024)
- obtain a scale factor and use it to solve scaling problems involving the calculation of surface areas and volumes of similar solids. (ACMGM025)

Unit 2

Unit Description

This unit has three topics: ‘Univariate data analysis and the statistical investigation process’, ‘Linear equations and their graphs’; and ‘Applications of trigonometry’.

‘ Univariate data analysis and the statistical investigation process’ develops students’ ability to organise and summarise univariate data in the context of conducting a statistical investigation.

‘Linear equations and their graphs’ uses linear equations and straight-line graphs, as well as linear-piecewise and step graphs, to model and analyse practical situations.

‘Applications of trigonometry’ extends students’ knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression and bearings in navigation.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in univariate data analysis and the statistical investigation process, linear equations and their graphs, and applications of trigonometry
- apply reasoning skills and solve practical problems in univariate data analysis and the statistical investigation process, linear equations and their graphs, and the applications of trigonometry
- implement the statistical investigation process in contexts requiring the analysis of univariate data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Content Descriptions

Topic 1: Univariate data analysis and the statistical investigation process

The statistical investigation process:

- review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results. (ACMGM026)

Making sense of data relating to a single statistical variable:

- classify a categorical variable as ordinal, such as income level (high, medium, low), or nominal, such as place of birth (Australia, overseas), and use tables and bar charts to organise and display the data (ACMGM027)
- classify a numerical variable as discrete, such as the number of rooms in a house, or continuous, such as the temperature in degrees Celsius (ACMGM028)
- with the aid of an appropriate graphical display (chosen from dot plot, stem plot, bar chart or histogram), describe the distribution of a numerical dataset in terms of modality (uni or multimodal), shape (symmetric versus positively or negatively skewed), location and spread and outliers, and interpret this information in the context of the data (ACMGM029)
- determine the mean and standard deviation of a dataset and use these statistics as measures of location and spread of a data distribution, being aware of their limitations. (ACMGM030)

Comparing data for a numerical variable across two or more groups:

- construct and use parallel box plots (including the use of the 'Q1 – 1.5 x IQR' and 'Q3 + 1.5 x IQR' criteria for identifying possible outliers) to compare groups in terms of location (median), spread (IQR and range) and outliers and to interpret and communicate the differences observed in the context of the data (ACMGM031)
- compare groups on a single numerical variable using medians, means, IQRs, ranges or standard deviations, as appropriate; interpret the differences observed in the context of the data; and report the findings in a systematic and concise manner (ACMGM032)
- implement the statistical investigation process to answer questions that involve comparing the data for a numerical variable across two or more groups; for example, are Year 11 students the fittest in the school? (ACMGM033)

Topic 2: Applications of trigonometry

Applications of trigonometry:

- review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle (ACMGM034)
- determine the area of a triangle given two sides and an included angle by using the rule $Area = \frac{1}{2} ab \sin C$, or given three sides by using Heron's rule, and solve related practical problems (ACMGM035)
- solve problems involving non-right-angled triangles using the sine rule (ambiguous case excluded) and the cosine rule (ACMGM036)
- solve practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of bearings in navigation. (ACMGM037)

Topic 3: Linear equations and their graphs

Linear equations:

- identify and solve linear equations (ACMGM038)
- develop a linear formula from a word description (ACMGM039)

Straight-line graphs and their applications:

- construct straight-line graphs both with and without the aid of technology (ACMGM040)
- determine the slope and intercepts of a straight-line graph from both its equation and its plot (ACMGM041)
- interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation (ACMGM042)
- construct and analyse a straight-line graph to model a given linear relationship; for example, modelling the cost of filling a fuel tank of a car against the number of litres of petrol required. (ACMGM043)

Simultaneous linear equations and their applications:

- solve a pair of simultaneous linear equations, using technology when appropriate (ACMGM044)
- solve practical problems that involve finding the point of intersection of two straight-line graphs; for example, determining the break-even point where cost and revenue are represented by linear equations. (ACMGM045)

Piece-wise linear graphs and step graphs:

- sketch piece-wise linear graphs and step graphs, using technology when appropriate (ACMGM046)
- interpret piece-wise linear and step graphs used to model practical situations; for example, the tax paid as income increases, the change in the level of water in a tank over time when water is drawn off at different intervals and for different periods of time, the charging scheme for sending parcels of different weights through the post. (ACMGM047)

Units 1 and 2 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics, in routine and non-routine problems in a variety of contexts • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models to solve routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics, in routine and non-routine problems • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems • selects and applies mathematical and statistical models to routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics, that apply to routine problems • selects and applies techniques in mathematics and statistics to solve routine problems • applies mathematical and statistical models to routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics • uses simple techniques in mathematics and statistics in routine problems • demonstrates familiarity with mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	• demonstrates limited familiarity with simple concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical or statistical models • uses digital technologies for arithmetic calculations and to display limited mathematical and statistical information

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments which are succinct and reasoned using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments which are clear and reasoned using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	• represents simple mathematical or statistical information in a structured context • communicates simple mathematical or statistical information • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

Unit 3

Unit Description

This unit has three topics: ‘Bivariate data analysis’, ‘Growth and decay in sequences’ and ‘Graphs and networks’.

‘Bivariate data analysis’ introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process.

‘Growth and decay in sequences’ employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population, or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4.

‘Graphs and networks’ introduces students to the language of graphs and the ways in which graphs, represented as a collection of points and interconnecting lines, can be used to model and analyse everyday situations such as a rail or social network.

Classroom access to technology to support the graphical and computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in bivariate data analysis, growth and decay in sequences, and graphs and networks
- apply reasoning skills and solve practical problems in bivariate data analysis, growth and decay in sequences, and graphs and networks
- implement the statistical investigation process in contexts requiring the analysis of bivariate data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Content Descriptions

Topic 1: Bivariate data analysis

The statistical investigation process:

- review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results. (ACMGM048)

Identifying and describing associations between two categorical variables:

- construct two-way frequency tables and determine the associated row and column sums and percentages (ACMGM049)
- use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association (ACMGM050)
- describe an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data. (ACMGM051)

Identifying and describing associations between two numerical variables:

- construct a scatterplot to identify patterns in the data suggesting the presence of an association (ACMGM052)
- describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak) (ACMGM053)
- calculate and interpret the correlation coefficient (r) to quantify the strength of a linear association. (ACMGM054)

Fitting a linear model to numerical data:

- identify the response variable and the explanatory variable (ACMGM055)
- use a scatterplot to identify the nature of the relationship between variables (ACMGM056)
- model a linear relationship by fitting a least-squares line to the data (ACMGM057)
- use a residual plot to assess the appropriateness of fitting a linear model to the data (ACMGM058)
- interpret the intercept and slope of the fitted line (ACMGM059)
- use the coefficient of determination to assess the strength of a linear association in terms of the explained variation (ACMGM060)
- use the equation of a fitted line to make predictions (ACMGM061)
- distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation (ACMGM062)
- write up the results of the above analysis in a systematic and concise manner. (ACMGM063)

Association and causation:

- recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between them (ACMGM064)
- identify possible non-causal explanations for an association, including coincidence and confounding due to a common response to another variable, and communicate these explanations in a systematic and concise manner. (ACMGM065)

The data investigation process:

- implement the statistical investigation process to answer questions that involve identifying, analysing and describing associations between two categorical variables or between two numerical variables; for example, is there an association between attitude to capital punishment (agree with, no opinion, disagree with) and sex (male, female)? is there an association between height and foot length? (ACMGM066)

Topic 2: Growth and decay in sequences

The arithmetic sequence:

- use recursion to generate an arithmetic sequence (ACMGM067)
- display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations (ACMGM068)
- deduce a rule for the *n*th term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions (ACMGM069)
- use arithmetic sequences to model and analyse practical situations involving linear growth or decay; for example, analysing a simple interest loan or investment, calculating a taxi fare based on the flag fall and the charge per kilometre, or calculating the value of an office photocopier at the end of each year using the straight-line method or the unit cost method of depreciation. (ACMGM070)

The geometric sequence:

- use recursion to generate a geometric sequence (ACMGM071)
- display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations (ACMGM072)
- deduce a rule for the *n*th term of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions (ACMGM073)
- use geometric sequences to model and analyse (numerically, or graphically only) practical problems involving geometric growth and decay; for example, analysing a compound interest loan or investment, the growth of a bacterial population that doubles in size each hour, the decreasing height of the bounce of a ball at each bounce; or calculating the value of office furniture at the end of each year using the declining (reducing) balance method to depreciate. (ACMGM074)

Sequences generated by first-order linear recurrence relations:

- use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form (ACMGM075)
- recognise that a sequence generated by a first-order linear recurrence relation can have a long term increasing, decreasing or steady-state solution (ACMGM076)
- use first-order linear recurrence relations to model and analyse (numerically or graphically only) practical problems; for example, investigating the growth of a trout population in a lake recorded at the end of each year and where limited recreational fishing is permitted, or the amount owing on a reducing balance loan after each payment is made. (ACMGM077)

Topic 3: Graphs and networks

The definition of a graph and associated terminology:

- explain the meanings of the terms: graph, edge, vertex, loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph (digraph), arc, weighted graph, and network (ACMGM078)
- identify practical situations that can be represented by a network, and construct such networks; for example, trails

connecting camp sites in a National Park, a social network, a transport network with one-way streets, a food web, the results of a round-robin sporting competition (ACMGM079)

- construct an adjacency matrix from a given graph or digraph. (ACMGM080)

Planar graphs:

- explain the meaning of the terms: planar graph, and face (ACMGM081)
- apply Euler's formula, $v + f - e = 2$, to solve problems relating to planar graphs. (ACMGM082)

Paths and cycles:

- explain the meaning of the terms: walk, trail, path, closed walk, closed trail, cycle, connected graph, and bridge (ACMGM083)
- investigate and solve practical problems to determine the shortest path between two vertices in a weighted graph (by trial-and-error methods only) (ACMGM084)
- explain the meaning of the terms: Eulerian graph, Eulerian trail, semi-Eulerian graph, semi-Eulerian trail and the conditions for their existence, and use these concepts to investigate and solve practical problems; for example, the Königsberg Bridge problem, planning a garbage bin collection route (ACMGM085)
- explain the meaning of the terms: Hamiltonian graph and semi-Hamiltonian graph, and use these concepts to investigate and solve practical problems; for example, planning a sight-seeing tourist route around a city, the travelling-salesman problem (by trial-and-error methods only). (ACMGM086)

Unit 4

Unit Description

This unit has three topics: 'Time series analysis'; ' Loans, investments and annuities' and 'Networks and decision mathematics'.

'Time series analysis' continues students' study of statistics by introducing them to the concepts and techniques of time series analysis. The content is to be taught within the framework of the statistical investigation process.

'Loans and investments and annuities' aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments.

'Networks and decision mathematics' uses networks to model and aid decision making in practical situations.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in time series analysis; loans, investments and annuities; and networks and decision mathematics
- apply reasoning skills and solve practical problems in time series analysis; loans, investments and annuities; and networks and decision mathematics
- implement the statistical investigation process in contexts requiring the analysis of time series data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Content Descriptions

Topic 1: Time series analysis

Describing and interpreting patterns in time series data:

- construct time series plots (ACMGM087)
- describe time series plots by identifying features such as trend (long term direction), seasonality (systematic, calendar-related movements), and irregular fluctuations (unsystematic, short term fluctuations), and recognise when there are outliers; for example, one-off unanticipated events. (ACMGM088)

Analysing time series data:

- smooth time series data by using a simple moving average, including the use of spreadsheets to implement this process (ACMGM089)
- calculate seasonal indices by using the average percentage method (ACMGM090)
- deseasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process (ACMGM091)
- fit a least-squares line to model long-term trends in time series data. (ACMGM092)

The data investigation process:

- implement the statistical investigation process to answer questions that involve the analysis of time series data. (ACMGM093)

Topic 2: Loans, investments and annuities

Compound interest loans and investments:

- use a recurrence relation to model a compound interest loan or investment, and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment (ACMGM094)
- calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)
- with the aid of a calculator or computer-based financial software, solve problems involving compound interest loans or investments; for example, determining the future value of a loan, the number of compounding periods for an investment to exceed a given value, the interest rate needed for an investment to exceed a given value. (ACMGM096)

Reducing balance loans (compound interest loans with periodic repayments):

- use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan (ACMGM097)
- with the aid of a financial calculator or computer-based financial software, solve problems involving reducing balance loans; for example, determining the monthly repayments required to pay off a housing loan. (ACMGM098)

Annuities and perpetuities (compound interest investments with periodic payments made from the investment):

- use a recurrence relation to model an annuity, and investigate (numerically or graphically) the effect of the amount invested, the interest rate, and the payment amount on the duration of the annuity (ACMGM099)
- with the aid of a financial calculator or computer-based financial software, solve problems involving annuities (including

perpetuities as a special case); for example, determining the amount to be invested in an annuity to provide a regular monthly income of a certain amount. (ACMGM100)

Topic 3: Networks and decision mathematics

Trees and minimum connector problems:

- explain the meaning of the terms tree and spanning tree identify practical examples (ACMGM101)
- identify a minimum spanning tree in a weighted connected graph either by inspection or by using Prim’s algorithm (ACMGM102)
- use minimal spanning trees to solve minimal connector problems; for example, minimising the length of cable needed to provide power from a single power station to substations in several towns. (ACMGM103)

Project planning and scheduling using critical path analysis (CPA):

- construct a network to represent the durations and interdependencies of activities that must be completed during the project; for example, preparing a meal (ACMGM104)
- use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project (ACMGM105)
- use ESTs and LSTs to locate the critical path(s) for the project (ACMGM106)
- use the critical path to determine the minimum time for a project to be completed (ACMGM107)
- calculate float times for non-critical activities. (ACMGM108)

Flow networks

- solve small-scale network flow problems including the use of the ‘maximum-flow minimum- cut’ theorem; for example, determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank (the source) to a terminal (the sink). (ACMGM109)

Assignment problems

- use a bipartite graph and/or its tabular or matrix form to represent an assignment/ allocation problem; for example, assigning four swimmers to the four places in a medley relay team to maximise the team’s chances of winning (ACMGM110)
- determine the optimum assignment(s), by inspection for small-scale problems, or by use of the Hungarian algorithm for larger problems. (ACMGM111)

Units 3 and 4 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics in routine and non-routine problems in a variety of contexts • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models to routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics in routine and non-routine problems • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems • selects and applies mathematical and statistical models to routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics that apply to routine problems • selects and applies techniques in mathematics and statistics to solve routine problems • applies mathematical and statistical models to routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics. • uses simple techniques in mathematics and statistics in routine problems • demonstrates familiarity with mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	• demonstrates limited familiarity with simple concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics . • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical or statistical models • uses digital technologies for arithmetic calculations and to display limited mathematical and statistical information

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments which are succinct and reasoned using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments which are clear and reasoned using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of the results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	• represents simple mathematical or statistical information in a structured context • communicates simple mathematical or statistical information • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

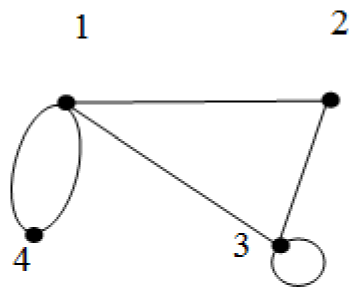
General Mathematics Glossary

Adjacency matrix

An adjacency matrix for a non-directed graph with n vertices is a $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j. In an adjacency matrix, a loop is counted as 1 edge.

Example:

Non-directed graph



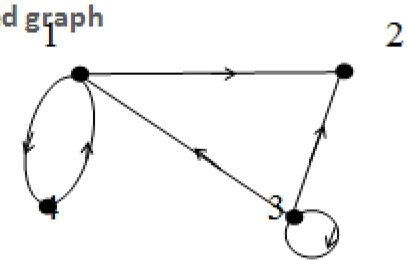
Adjacency matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

For a directed graph the entry in row i and column j is the number of directed edges (arcs) joining the vertex i and j in the direction i to j.

Example:

Directed graph



Adjacency matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Adjacent (graph)

see the definition of a graph on page 39

Algorithm

An algorithm is a precisely defined routine procedure that can be applied and systematically followed through to a conclusion. An example is Prim's algorithm for determining a minimum spanning tree in a network.

Arc see directed graph

Angle of depression

The angle a line makes below a horizontal plane.

Angle of elevation

The angle a line makes above a horizontal plane.

Annuity

An annuity is a compound interest investment from which payments are made on a regular basis for a fixed period of time. At the end of this time the investment has no residual value.

Area of a triangle

The general rule for determining the area of a triangle is: $area = \frac{1}{2}\ base\ \times\ height$

Arithmetic sequence

An arithmetic sequence is a sequence of numbers such that the difference between any two successive members of the sequence is constant.

For example, the sequence

2, 5, 8, 11, 14, 17, …

is an arithmetic sequence with first term 2 and common difference 3.

By inspection of the sequence, the rule for the nth term t_n of this sequence is:

$t_n = 2 + (n - 1)3 = 3n - 1\ n \geq 1$

If t_n is used to denote the nth term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2$, $t_{n+1} = t_n + 3\ n \geq 1$

Association

A general term used to describe the relationship between two (or more) variables. The term association is often used interchangeably with the term correlation. The latter tends to be used when referring to the strength of a linear relationship between two numerical variables.

Average percentage method

In the average percentage method for calculating a seasonal index, the data for each 'season' are expressed as percentages of the average for the year. The percentages for the corresponding 'seasons' for different years are then averaged using a mean or median to arrive at a seasonal index.

Bearings (compass and true)

A bearing is the direction of a fixed point, or the path of an object, from the point of observation.

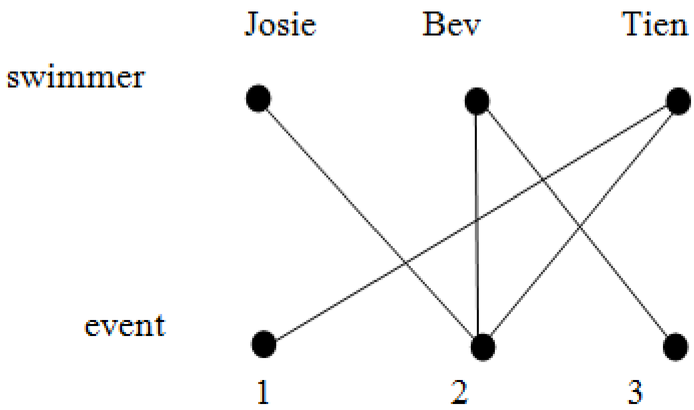
Compass bearings are specified as angles either side of north or south. For example a compass bearing of N50°E is found by facing north and moving through an angle of 50° to the East.

True (or three figure) bearings are measured in degrees from the north line. Three figures are used to specify the direction. Thus the direction of north is specified as 000°, east is specified as 090°, south is specified as 180° and north-west is specified as 315°.

Bipartite Graph

A bipartite graph is a graph whose set of vertices can be split into two distinct groups in such a way that each edge of the graph joins a vertex in the first group to a vertex in the second group.

Example:



Bridge see connected graph

Book value

The book value is the value of an asset recorded on a balance sheet. The book value is based on the original cost of the asset less depreciation.

For example, if the original cost of a printer is $500 and its value depreciates by $100 over the next year, then its book value at the end of the year is $400.

There are three commonly used methods for calculating yearly depreciation in the value of an asset, namely, reducing balancedepreciation, flat rate depreciation or unit cost depreciation.

Break-even point

The break-even point is the point at which revenue begins to exceed the cost of production.

Categorical data

Data associated with a categorical variable is called categorical data.

Categorical variable

A categorical variable is a variable whose values are categories.

Examples include blood group (A, B, AB or O) or house construction type (brick, concrete, timber, steel, other).

Categories may have numerical labels, eg. the numbers worn by player in a sporting team, but these labels have no numerical significance, they merely serve as labels.

Causation

A relationship between an explanatory and a response variable is said to be causal if the change in the explanatory variable actually causes a change in the response variable. Simply knowing that two variables are associated, no matter how strongly, is not sufficient evidence by itself to conclude that the two variables are causally related.

Possible explanations for an observed association between an explanatory and a response variable include:

the explanatory variable is actually causing a change in the response variable

there may be causation, but the change may also be caused by one or more uncontrolled variables whose effects cannot be disentangled from the effect of the response variable. This is known as confounding.

there is no causation, the association is explained by at least one other variable that is associated with both the explanatory and the response variable . This is known as a common response.

the response variable is actually causing a change in the explanatory variable

Closed path

See path

Closed trail

See trail

Closed walk

See walk

Coefficient of determination

In a linear model between two variables, the coefficient of determination (R^2) is the proportion of the total variation that can be explained by the linear relationship existing between the two variables, usually expressed as a percentage. For two variables only, the coefficient of determination is numerically equal to the square of the correlation coefficient (r^2).

Example

A study finds that the correlation between the heart weight and body weight of a sample of mice is $r = 0.765$. The coefficient of determination = $r^2 = 0.765^2 = 0.5852$ … or approximately 59%

From this information, it can be concluded that approximately 59% of the variation in heart weights of these mice can be explained by the variation in their body weights.

Note: The coefficient of determination has a more general and more important meaning in considering relationships between more than two variables, but this is not a school level topic.

Common response
See Causation

Complete graph

A complete graph is a simple graph in which every vertex is joined to every other vertex by an edge.

The complete graph with n vertices is denoted K_n .

Compound interest

The interest earned by investing a sum of money (the principal) is compound interest if each successive interest payment is added to the principal for the purpose of calculating the next interest payment.

For example, if the principal P earns compound interest at the rate of i % per period, then after n periods the total amount accrued is $P\left(1+\frac{i}{n}\right)^{n}$. When plotted on a graph, the total amount accrued is seen to grow exponentially.

Confounding

See Causation

Connected graph

A graph is connected if there is a path between each pair of vertices. A bridge is an edge in a connected graph that, if removed, leaves a graph disconnected.

Continuous data

Data associated with a continuous variable is called continuous data.

Continuous variable

A continuous variable is a numerical variable that can take any value that lies within an interval. In practice, the values taken are subject to accuracy of the measurement instrument used to obtain these values.

Examples include height, reaction time and systolic blood pressure.

Correlation

Correlation is a measure of the strength of the linear relationship between two variables. See also association.

Correlation coefficient (r)

The correlation coefficient (r) is a measure of the strength of the liner relationship between a pair of variables. The formula for calculating r is given below.

For variables x and y, and computed for n cases, the formula for r is:

$$r=\frac{1}{n-1}\ \sum\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right)$$

Cosine rule

For a triangle of side lengths a, b and c and angles A, B and C, the cosine rule states that

$c^2 = a^2 + b^2 - 2ab \cos C$

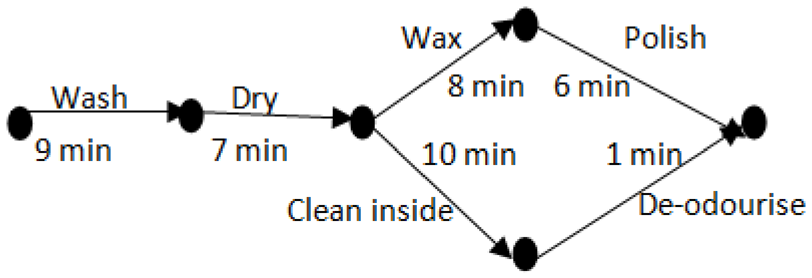
CPI

The Consumer Price Index (CPI) is a measure of changes, over time, in retail prices of a constant basket of goods and services representative of consumption expenditure by resident households in Australian metropolitan areas.

Critical path analysis (CPA)

A project often involves many related activities some of which cannot be started until one or more earlier tasks have been completed. One way of scheduling such activities that takes this into account is to construct a network diagram.

The network diagram below can be used to schedule the activities of two or more individuals involved in cleaning and polishing a car. The completion times for each activity are also shown.



Critical path analysis is a method for determining the longest path (the critical path) in such a network and hence the minimum time in which the project can be completed. There may be more that one critical path in the network. In this project the critical path is 'Wash-Dry-Wax-Polish' with a total completion time of 30 minutes.

The earliest starting time (EST) of an activity 'Polish' is 24 minutes because activities 'Wash', 'Dry' and 'Wax' must be completed first. The process of systematically determining earliest starting times is called forward scanning.

The shortest time that the project can be completed is 30 minutes. Thus, the latest starting time (LST) for the activity 'De-odourise' is 29 minutes. The process of systematically determining latest starting times is called backward scanning.

Float or slack

Is the amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks. For example, the activity 'De-odourise' is said to have a float of 3 minutes because its earliest EST (26 minutes) is three minutes before its LST (29 minutes). As a result this activity can be started at any time between 26 and 29 minutes after the project started. All activities on a critical path have zero floats.

Cut (in a flow network)

In a flow network, a cut is a partition of the vertices of a graph into two separate groups with the source in one group and the sink in the other.

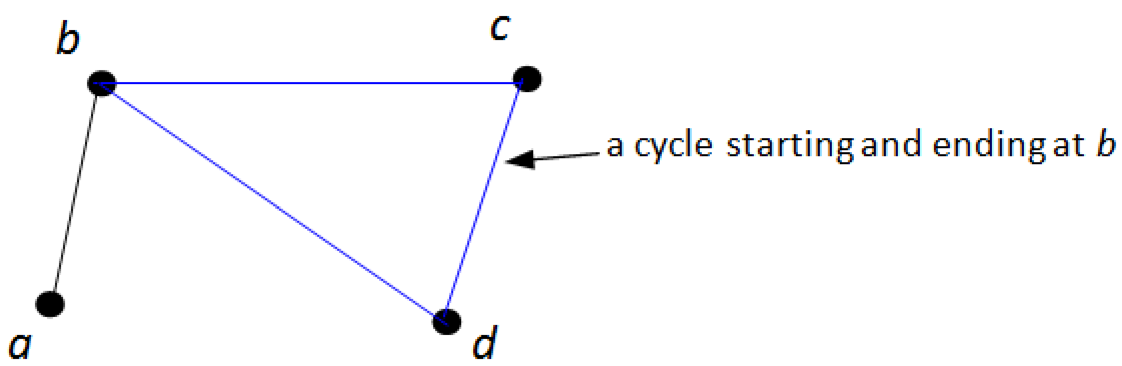
The capacity of the cut is the sum of the capacities of the cut edges directed from source to sink. Cut edges directed from sink to source are ignored.

Example:

$\genfrac{}{}{0pt}{}{2}{4}$, $\genfrac{}{}{0pt}{}{3}{4}$, $\genfrac{}{}{0pt}{}{1}{4}$, $\genfrac{}{}{0pt}{}{6}{4}$, $\genfrac{}{}{0pt}{}{3}{8}$, $\genfrac{}{}{0pt}{}{2}{2}$, $\genfrac{}{}{0pt}{}{3}{2}$

Cycle

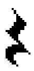
A cycle is a closed walk begins and starts at the same vertex and in which has no repeated edges or vertices except the first.. If *a*, *b*, *c* and *d* are the vertices of a graph, the closed walk *bcdb* that starts and ends at vertex *b* (shown in blue) an example of a cycle.



Degree of a vertex (graph)

In a graph, the degreeof a vertex is the number of edges incident with the vertex, with loops counted twice. It is denoted deg *v*.

In the graph below, deg *a* = 4, deg *b* = 2, deg *c* = 4 and deg *d*=2.



Digraph

See directed graph

Directed graph

A directed graph is a diagram comprising points, called vertices, joined by directed lines called arcs. The directed graphs are commonly called digraphs.

Discrete data

Discrete data is data associated with a discrete variable. Discrete data is sometimes called count data.

Discrete variable

A discrete variable is a numerical variable that can take only integer values.

Examples include the number of people in a car, the number of decayed teeth in 18 year-old males, etc.

Earliest starting time (EST)

See Critical Path Analysis

Edge

See graph

Effective annual rate of interest

The effective annual rate of interest $i_{\text{effective}}$ is used to compare the interest paid on loans (or investments) with the same nominal annual interest rate *i* but with different compounding periods (daily, monthly, quarterly, annually, other)

If the number of compounding periods per annum is *n*, then $i_{effective} = \left(1 + \frac{i}{n}\right)^n - 1$

For example if the quoted annual interest rate for a loan is 9%, but interest is charged monthly, then the effective annual interest rate charged is $i_{effective} = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 0.9416$, or around 9.4%.

Diminishing value depreciation see Reducing balance depreciation

Elements (Entries) of a matrix

The symbol a_{ij} represents the (*i,j*) element occurring in the i^{th} row and the j^{th} column.

For example a general 3 × 2 matrix is:

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ where a_{32} is the element in the third row and the second column

Euler's formula

For a connected planar graph, Euler's rule states that

$v + f - e = 2$

where *v* is the number vertices, *e* the number of edges and *f* is the number of faces.

Eulerian

A connected graph is Eularian if it has a closed trail (starts and ends at the same vertex), that is, includes every edge and once only; such a trail is called an Eulerian trail. An Eularian trial may include repeated vertices. A connected graph is semi-Eularian if there is an open trail that includes every once only.

Explanatory variable

When investigating relationships in bivariate data, the explanatory variable is the variable used to explain or predict a difference in the response variable.

For example, when investigating the relationship between the temperature of a loaf of bread and the time it has spent in a hot oven, *temperature* is the response variable and *time* is the explanatory variable.

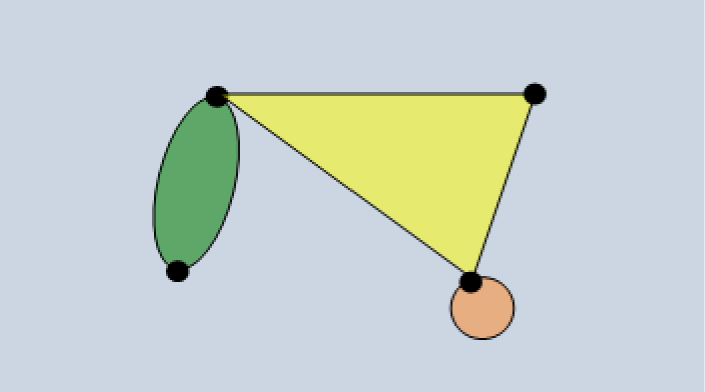
Extrapolation

In the context of fitting a linear relationship between two variables, extrapolation occurs when the fitted model is used to make predictions using values of the explanatory variable that are outside the range of the original data. Extrapolation is a dangerous process as it can sometimes lead to quite erroneous predictions.

See also interpolation.

Face

The faces of a planar graph are the regions bounded by the edges including the outer infinitely large region. The planar graph shown has four faces.



First-order linear recurrence relation

A first-order linear recurrence relation is defined by the rule: $t_0 = a$,$t_{n+1} = bt_n + c$ for $n \geq 1$

For example, the rule: $t_0 = 10$, $t_n = 5t_{n-1} + 1$ for $n \geq 1$ is a first-order recurrence relation.

The sequence generated by this rule is: 10, 51, 256, … as shown below.

$t_1 = 10$, $t_2 = 5t_1 + 1 = 5 \times 10 + 1 = 51$, $t_3 = 5t_2 + 1 = 5 \times 51 + 1 = 256$, …

Five-number summary

A five-number summary is a method of summarising a set of data using the minimum value, the lower or first-quartile (Q_1), the median, the upper or third-quartile (Q_3) and the maximum value. Forms the basis for a boxplot.

Flat rate depreciation

In flat rate or straight-line depreciation the value of an asset is depreciated by a fixed amount each year. Usually this amount is specified as a fixed percentage of the original cost.

Float time

See Critical Path Analysis

Flow network

A flow network is a directed graph where each edge has a capacity (e.g. 100 cars per hour, 800 litres per minute, etc) and each edge receives a flow. The amount of flow on an edge cannot exceed the capacity of the edge. A flow must satisfy the restriction that the amount of flow into a node equals the amount of flow out of it, except when it is a source, which has more outgoing flow, or a sink, which has more incoming flow. A flow network can be used to model traffic in a road system, fluids in pipes, currents in an electrical circuit, or any situation in which something travels through a network of nodes.

Food web

A food web (or food chain) depicts feeding connections (who eats whom) in an ecological community.

Geometric growth or decay (sequence)

A sequence displays geometric growth or decay when each term is some constant multiple (greater or less than one) of the preceding term. A multiple greater than one corresponds to growth. A multiple less than one corresponds to decay.

For example, the sequence:

1, 2, 4, … displays geometric growth because each term is double the previous term.

100, 10, 0.1, … displays geometric decay because each term is one tenth of the previous term.

Geometric growth is an example of exponential growth in discrete situations.

Geometric sequence

A geometric sequence, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed non-zero number called the common ratio. For example, the sequence

2, 6, 18, ...

is a geometric sequence with first term 2 and common ratio 3.

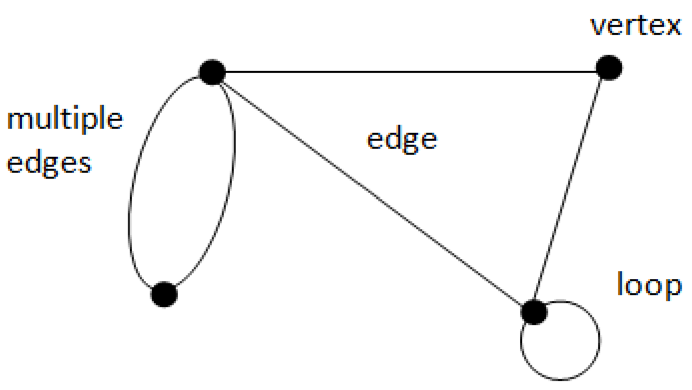
By inspection of the sequence, the rule for the *n*th term of this sequence is:

$t_n = 2 \times 3^{n-1} n \geq 1$

If t_n is used to denote the *n*th term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2$, $t_{n+1} = 3t_n \; n \geq 1$

Graph

A graph is a diagram that consists of a set of points, called vertices that are joined by a set of lines called edges. Each edge joins two vertices. A loop is an edge in a graph that joins a vertex in a graph to itself. Two vertices are adjacent if they a joined by an edge. Two or more edges connect the same vertices are called multiple edges.



GST

The GST (Goods and Services Tax) is a broad sales tax of 10% on most goods and services transactions in Australia.

Hamiltonian

A Hamiltonian cycle is a cycle that includes each vertex in a graph (except the first), once only.
A Hamilton path is path that includes every vertex in a graph once only. A Hamilton path that begins and ends at the same vertex is a Hamiltonian cycle.

Heron's rule

Heron's rule is a rule for determining the area of a triangle given the lengths of its sides.

The area *A* of a triangle of side lengths *a*, *b* and *c* is given by

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $= \frac{1}{2}\left(a+b+c\right)$.

Hungarian algorithm

The Hungarian algorithm is used to solve assignment (allocation) problems.

Identity matrix

A multiplicative identity matrix is a square matrix in which all of the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter *I*.

For example,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ are both identity matrices.

.

There is an identity matrix for each size (or order) of a square matrix. When clarity is needed, the order is written with a subscript: I_n

Interpolation

In the context of fitting a linear relationship between two variables, interpolation occurs when the fitted model is used to make predictions using values of the explanatory variable that lie within the range of the original data.

See also extrapolation.

Inverse of a 2 × 2 matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Is

$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ provided $ad - bc \neq 0$

Inverse of a square matrix

The inverse of a square matrix*A* is written as A^{-1} and has the property that

$AA^{-1} = A^{-1}A = I$

Not all square matrices have an inverse. A matrix that has an inverse is said to be invertible.

Irregular variation or noise (time series)

Irregular variation or noise is erratic and short-term variation in a time series that is the product of chance occurrences.

Königsberg Bridge problem

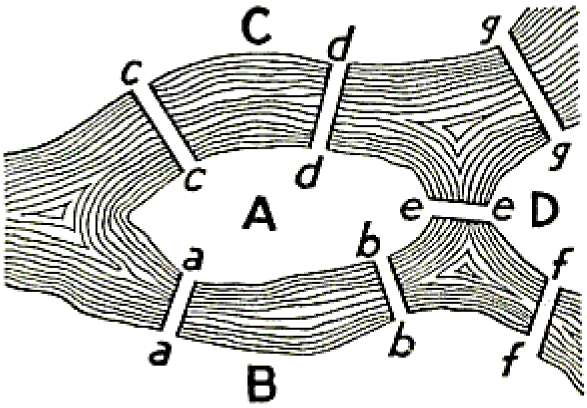


FIGURE 98. *Geographic Map: The Königsberg Bridges.*

The Königsberg bridge problem asks: Can the seven bridges of the city of Königsberg all be traversed in a single trip that starts and finishes at the same place?

Latest starting time (LST)

See Critical Path Analysis

Leading diagonal

The leading diagonal of a square matrix is the diagonal that runs from the top left corner to the bottom right corner of the matrix.

Least-squares line

In fitting a straight-line $y = a + bx$ to the relationship between a response variable y and an explanatory variable x, the least-squares line is the line for which the sum of the squared residuals is the smallest.

The formula for calculating the slope (b) and the intercept (a) of the least squares line is given below.

For variables x and y computed for n cases, the slope (b) and intercept (a) of the least-squares line are given by:

$$b = \frac{\sum(x_i - \overline{x})(y_i - \overline{y})}{\sum(x_i - \overline{x})^2} \quad \text{or} \quad b = r\,\frac{s_y}{s_x} \quad \text{and} \quad a = \overline{y} - b\overline{x}$$

Length (of a walk)

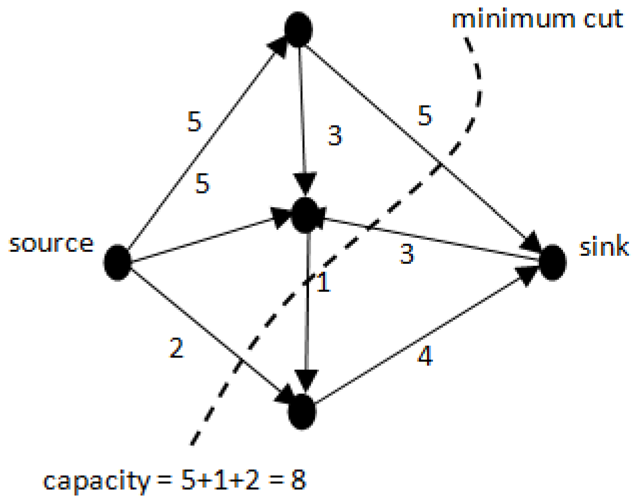
The length of a walk is the number of edges it includes.

Minimum cut-maximum flow theorem

The maximum flow–minimum cut theorem states that in a flow network, the maximum flow from the source to the sink is equal to the capacity of the minimum cut.

In everyday language, the minimum cut involves identifies the 'bottle-neck' in the system.

Example:



Linear equation

A linear equation in one variable x is an equation of the form $ax + b = 0$, e.g. $3x + 1 = 0$

A linear equation in two variables x and y is an equation of the form $ax + by + c = 0$,

e.g. $2x – 3y + 5 = 0$

Linear graph

A linear graph is a graph of a linear equation with two variables. If the linear equation is written in the form $y = a + bx$, then a represents the y-intercept and b represents the slope (or gradient) of the linear graph.

Linear growth or decay (sequence)

A sequence displays linear growth or decay when the difference between successive terms is constant. A positive constant difference corresponds to linear growth while a negative constant difference corresponds to decay.

Examples:

The sequence, 1, 4, 7, … displays linear growth because the difference between successive terms is 3.

The sequence, 100, 90, 80, … displays linear decay because the difference between successive terms is –10. By definition, arithmetic sequences display linear growth or decay.

Location

The notion of central or 'typical value' in a sample distribution.

See also mean, median and mode.

Matrix (matrices)

A matrix is a rectangular array of elements or entities displayed in rows and columns.

For example,

$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ are both matrices with six elements.

Matrix A is said to be a 3×2 matrix (three rows and two columns) while B is said to be a 2 × 3 matrix (two rows and three columns).

A square matrix has the same number of rows and columns.

A column matrix (or vector) has only one column.

A row matrix (or vector) has only one row.

Matrix multiplication

Matrix multiplication is the process of multiplying a matrix by another matrix.

For example, forming the product

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 25 \\ 11 & 45 \end{bmatrix}$$

The multiplication is defined by 1 × 2 + 8 × 0 + 0 × 4 = 2

1 × 1 + 8 × 3 + 0 × 4 = 25

2 × 2 + 5 × 0 + 7 × 1 = 11

2 × 1 + 5 × 3 + 7 × 4 = 45

This is an example of the process of matrix multiplication.

The product AB of two matrices A and B of size $m \times n$ and $p \times q$ respectively is defined if $n = p$.

If $n = p$ the resulting matrix has size $m \times q$.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & ba_{23} \end{bmatrix}$ then

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{12}b_{12} + a_{22}b_{22} & a_{12}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$$

Order (of a matrix)

See size (of a matrix)

Scalar multiplication (matrices)

Scalar multiplication is the process of multiplying a matrix by a scalar (number).

For example, forming the product

$$10 \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 0 & 30 \\ 10 & 40 \end{bmatrix}$$

is an example of the process of scalar multiplication.

In general for the matrix A with elements a_{ij} the elements of kA are ka_{ij}.

Mean

The arithmetic mean of a list of numbers is the sum of the data values divided by the number of values in the list.

In everyday language, the arithmetic mean is commonly called the average.

For example, for the following list of five numbers 2, 3, 3, 6, 8 the mean equals

$$\frac{2+3+3+6+8}{5} = \frac{22}{5} = 4.4$$

In more general language, the mean of n observations $x_1, x_2, \ldots, x_n$ is $\overline{x} = \frac{\sum x_i}{n}$

Median

The median is the value in a set of ordered set of data values that divides the data into two parts of equal size. When there are an odd number of data values, the median is the middle value. When there is an even number of data values, the median is the average of the two central values.

Minimum spanning tree

For a given connected weighted graph, the minimum spanning tree is the spanning tree of minimum length.

Multiple edges

See graph

Mode

The mode is the most frequently occurring value is a data set.

Moving average

In a time series, a simple moving average is a method used to smooth the time series whereby each observation is replaced by a simple average of the observation and its near neighbours. This process reduces the effect of non-typical data and makes the overall trend easier to see.

Note: There are times when it is preferable to use a weighted average rather simple average, but this is not required in the current curriculum.

Network

The word network is frequently used in everyday life, e.g. television network, rail network, etc. Weighted graphs or digraphs can often be used to model such networks.

Open path

See path

Open trail

See trail

Open walk

See walk

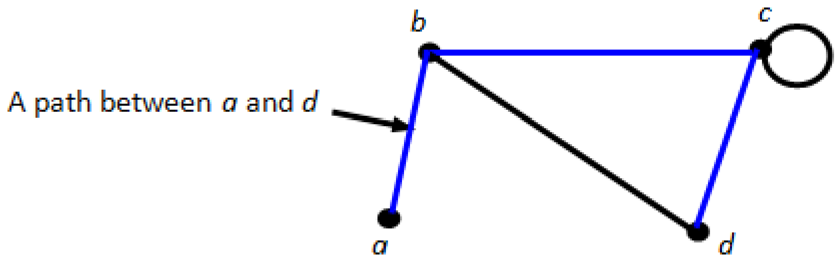
Outlier

An outlier in a set of data is an observation that appears to be inconsistent with the remainder of that set of data. An outlier is a surprising observation.

Path (in a graph)

A path in a graph is a walk in which all of the edges and all the vertices are different.. A path that starts and finishes at different vertices is said to be open, while a path that stats and finishes at the same vertex is said to be closed. A cycle is a closed path.

If *a* and *d* are the vertices of a graph, a walk from *a* to *d* along the edges coloured blue is a path. Depending on the graph, there may be multiple paths between the same two vertices, as is the case here.



Perpetuity

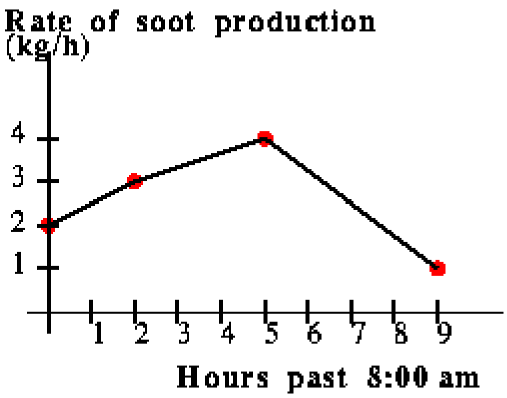
A perpetuity is a compound interest investment from which payments are made on a regular basis in perpetuity (forever). This is possible because the payments made at the end of each period exactly equal the interest earned during that period.

Piecewise-linear graph

A graph consisting of one or more none overlapping line segments.

Sometimes called a line segment graph.

Example:



Planar graph

A planar graph is a graph that can be drawn in the plane. A planar graph can always be drawn so that no two edges cross.

Price to earnings ratio (of a share)

The price to earnings ratio of a share (P/E ratio) is defined as :

$$P/Eratio = \frac{Market\ price\ per\ share}{Annual\ earnings\ per\ share}$$

Prim’s algorithm

An algorithm for determining a minimum spanning tree in a connected weighted graph.

Recurrence relation

A recurrence relation is an equation that recursively defines a sequence; that is, once one or more initial terms are given, each further term of the sequence is defined as a function of the preceding terms.

Recursion

See recurrence relation

Reducing balance depreciation

In reducing balance depreciation the value of an asset is depreciated by a fixed percentage of its value each year.

Reducing balance depreciation is sometimes called diminishing value depreciation.

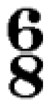
Reducing balance loan

A reducing balance loan is a compound interest loan where the loan is repaid by making regular payments and the interest paid is calculated on the amount still owing (the reducing balance of the loan) after each payment is made.

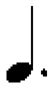
Residual plot

A residual plot is a scatterplot with the residual values shown on the vertical axis and the explanatory variable shown on the horizontal axis. Residual plots are useful in assessing the fit of the statistical model (e.g., by a least-squares line).

When the least-squares line captures the overall relationship between the response variable *y* and the explanatory variable *x*, the residual plot will have no clear pattern (be random) see opposite. This is what is hoped for.



scatterplot with least squares line



residual plot

If the least-squares line fails to capture the overall relationship between a response variable and an explanatory variable, a residual plot will reveal a pattern in the residuals. A residual plot will also reveal any outliers that may call into question the use of a least-squares line to describe the relationship. Interpreting patterns in residual plots is a skilled art and is not required in this curriculum.

Residual values

The difference between the observed value and the value predicted by a statistical model (e.g., by a least-squares line)

Response variable

See Explanatory variable

Round-robin sporting competition

A single round robin sporting competition is a competition in which each competitor plays each other competitor once only.

Scale factor

A scale factor is a number that scales, or multiplies, some quantity. In the equation $y = kx$, k is the scale factor for x.

If two or more figures are similar, their sizes can be compared. The scale factor is the ratio of the length of one side on one figure to the length of the corresponding side on the other figure. It is a measure of magnification, the change of size.

Scatterplot

A two-dimensional data plot using Cartesian co-ordinates to display the values of two variables in a bivariate data set.

For example the scatterplot below displays the CO_2 emissions in tonnes per person (*co2*) plotted against Gross Domestic Product per person in $US (*gdp*) for a sample of 24 countries in 2004. In constructing this scatterplot, gdp has been used as the explanatory variable.

Seasonal adjustment (adjusting for seasonality)

A term used to describe a time series from which periodic variations due to seasonal effects have been removed.

See also seasonal index.

Seasonal index

The seasonal index can be used to remove seasonality from data. An index value is attached to each period of the time series within a year. For the seasons of the year (Summer, Autumn, Winter, Spring) there are four separate seasonal indices; for months, there are 12 separate seasonal indices, one for each month, and so on. There are several methods for determining seasonal indices.

Seasonal variation

A regular rise and fall in the time series that recurs each year.

Seasonal variation is measured in terms of a seasonal index.

Smoothing (time series) see moving average

Semi-Eularian graph

See Eularian graph

Sequence

A sequence is an ordered list of numbers (or objects).

For example 1, 3, 5, 7 is a sequence of numbers that differs from the sequence 3, 1, 7, 5 as order matters.

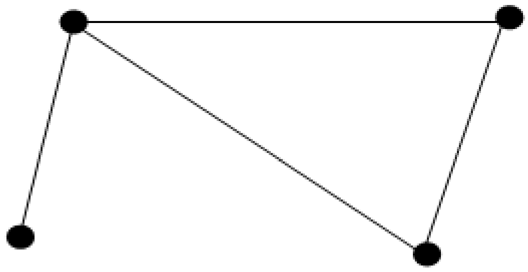
A sequence maybe finite, for example, 1, 3, 5, 7 (the sequence of the first four odd numbers), or infinite, for example, 1, 3, 5, … (the sequence of all odd numbers).

Similar figures

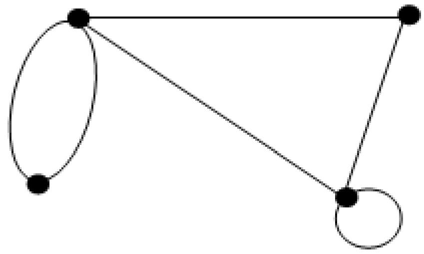
Two geometric figures are similar if they are of the same shape but not necessarily of the same size.

Simple graph

A simple graph has no loops or multiple edges.



simple graph



non-simple graph

Simple interest

Simple interest is the interest accumulated when the interest payment in each period is a fixed fraction of the principal. For example, if the principle P earns simple interest at the rate of i % per period, then after n periods the accumulated simple interest is $nP\,\frac{i}{100}$

When plotted on a graph, the total amount accrued is seen to grow linearly.

Sine rule

For a triangle of side lengths a, b and c and angles A, B and C, the sine rule states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Singular matrix

A matrix is singular if det A = 0. A singular matrix does not have a multiplicative inverse.

Size (of a matrix)

Two matrices are said to have the same size (or order) if they have the same number of rows and columns. A matrix with m rows and n columns is said to be a $m \times n$ matrix.

For example, the matrices

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \textit{ and } \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

have the same size. They are both 2 × 3 matrices.

Slope (gradient)

The slope or gradient of a line describes its steepness, incline, or grade.

Slope is normally described by the ratio of the "rise" divided by the "run" between two points on a line.

See also linear graph.

Spanning tree

A spanning tree is a subgraph of a connected graph that connects all vertices and is also a tree.

Standard deviation

The standard deviation is a measure of the variability or spread of a data set. It gives an indication of the degree to which the individual data values are spread around their mean.

The standard deviation of n observations $x_1, x_2, \ldots, x_n$ is

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

Statistical investigation process

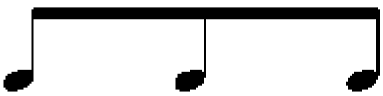
The statistical investigation process is a cyclical process that begins with the need to solve a real world problem and aims to reflect the way statisticians work. One description of the statistical investigation process in terms of four steps is as follows.

Step 1. Clarify the problem and formulate one or more questions that can be answered with data.

Step 2. Design and implement a plan to collect or obtain appropriate data.

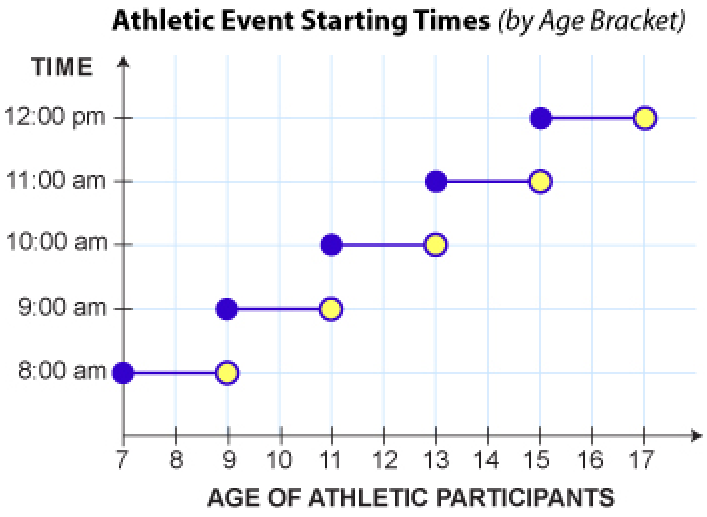
Step 3. Select and apply appropriate graphical or numerical techniques to analyse the data.

Step 4. Interpret the results of this analysis and relate the interpretation to the original question; communicate findings in a systematic and concise manner.



Step graph

A graph consisting of one or more non-overlapping horizontal line segments that follow a step-like pattern.



Matrices

Addition of matrices

If A and B are matrices of the same size (order) and the elements of A are a_{ij} and the elements of B are b_{ij} then the elements of $A + B$ are $a_{ij} + b_{ij}$

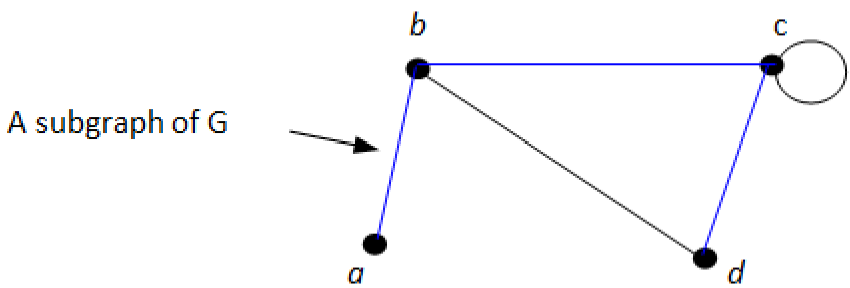
For example if $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 6 \end{bmatrix}$ Then $A + B = \begin{bmatrix} 7 & 2 \\ 2 & 4 \\ 2 & 10 \end{bmatrix}$

Straight-line depreciation

See: flat rate depreciation

Subgraph

When the vertices and edges of a graph A (shown in blue) are the vertices and edges of the graph G, graph A is said to be a subgraph of graph G.



Time series

Values of a variable recorded, usually at regular intervals, over a period of time. The observed movement and fluctuations of many such series comprise long-term trend, seasonal variation, and irregular variation or noise.

Time series plot

The graph of a time series with time plotted on the horizontal axis.

Trail

A trail is a walk in which no edge is repeated.

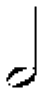
The travelling salesman problem

The travelling salesman problem can be described as follows: Given a list of cities and the distance between each city, find the shortest possible route that visits each city exactly once.

While in simple cases this problem can be solved by systematic identification and testing of possible solutions, no there is no known efficient method for solving this problem.

Tree

A tree is a connected graph with no cycles.



Trend (time series)

Trend is the term used to describe the general direction of a time series (increasing/ decreasing) over a long period of time.

Triangulation

The process of determining the location of a point by measuring angles to it from known points at either end of a fixed baseline, rather than measuring distances to the point directly. The point can then be fixed as the third point of a triangle with one known side and two known angles.

Two-way frequency table

A two-way frequency table is commonly used for displaying the two-way frequency distribution that arises when a group of individuals or objects are categorised according to two criteria.

For example, the two-way table below displays the frequency distribution that arises when 27 children are categorised according to *hair type* (straight or curly) and *hair colour* (red, brown, blonde, black).

Hair colour	Hair type		Total
	Straight	Curly	
red	1	1	2
brown	8	4	12
blonde	1	3	4
black	7	2	9
Total	17	10	27

The row and column totals represent the total number of observations in each row and column and are sometimes called row sums or column sums.

If the table is ‘percentaged’ using row sums the resulting percentages are called row percentages. If the table is ‘percentaged’ using row sums the resulting percentages are called column percentages.

Unit cost depreciation

In unit cost depreciation, the value of an asset is depreciated by an amount related to the number of units produced by the asset during the year.

Geometry and trigonometry

Vertex

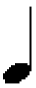
See graph

Walk (in a graph)

A walk in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. A walk that starts and finishes at different vertices is said to be an openwalk. A walk that starts and finishes at the same vertex is said to be closed walk.

If *a*, *b*, *c* and *d* are the vertices of a graph with edges *ab*, *bc*, *cc*, *cd* and *bd*, then the sequence of edges (*ab*, *bc*, *cc*, *cd*) constitute a walk. The route followed on this walk is shown in blue on the graph below.

This walk is denoted by the sequence of vertices *abccd*. The walk is open because it begins and finishes at different vertices.



A walk can include repeated vertices (as is the case above) or repeated edges.

A example of a closed walk with both repeated edges and hence vertices is defined by the sequence of edges (*ab*, *bd*, *db*, *ba*) and is denoted by the sequence of vertices *abdba*. The route followed is shown in red in the graph below.

Depending on the graph, there may be multiple walks between the same two vertices, as is the case here.

Weighted graph

A weighted graph is a graph in which each edge is labelled with a number used to represent some quantity associated with the edge. For example, if the vertices represent towns, the weights on the edges may represent the distances in kilometres between the towns.

Zero matrix

A zero matrix is a matrix if all of its entries are zero. For example:

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ And $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ are zero matrices.

Statistics

The Australian Curriculum
Mathematical Methods

Mathematical Methods

Rationale and Aims

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real-world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The major themes of Mathematical Methods are calculus and statistics. They include as necessary prerequisites studies of algebra, functions and their graphs, and probability. They are developed systematically, with increasing levels of sophistication and complexity. Calculus is essential for developing an understanding of the physical world because many of the laws of science are relationships involving rates of change. Statistics is used to describe and analyse phenomena involving uncertainty and variation. For these reasons this subject provides a foundation for further studies in disciplines in which mathematics and statistics have important roles. It is also advantageous for further studies in the health and social sciences. In summary, the subject Mathematical Methods is designed for students whose future pathways may involve mathematics and statistics and their applications in a range of disciplines at the tertiary level.

For all content areas of Mathematical Methods, the proficiency strands of the F-10 curriculum are still applicable and should be inherent in students' learning of this subject. These strands are Understanding, Fluency, Problem solving and Reasoning, and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as calculating derivatives and integrals, or solving quadratic equations, and frees up working memory for more complex aspects of problem solving. The ability to transfer skills to solve problems based on a wide range of applications is a vital part of mathematics in this subject. Because both calculus and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout this subject.

Mathematical Methods is structured over four units. The topics in Unit 1 build on students' mathematical experience. The topics 'Functions and graphs', 'Trigonometric functions' and 'Counting and probability' all follow on from topics in the F-10 curriculum from the strands, Number and Algebra, Measurement and Geometry and Statistics and Probability. In Mathematical Methods there is a progression of content and applications in all areas. For example, in Unit 2 differential calculus is introduced, and then further developed in Unit 3 where integral calculus is introduced. Discrete probability distributions are introduced in Unit 3, and then continuous probability distributions and an introduction to statistical inference conclude Unit 4.

Aims

Mathematical Methods aims to develop students':

- understanding of concepts and techniques drawn from algebra, the study of functions, calculus, probability and statistics
- ability to solve applied problems using concepts and techniques drawn from algebra, functions, calculus, probability and statistics
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Organisation

Overview of senior secondary Australian Curriculum

ACARA has developed senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications. The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

The senior secondary Australian Curriculum specifies content and achievement standards for each senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) expected of students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment and examinations). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years.

State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into their courses. They will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and Board. They will determine the assessment and certification specifications for their local courses that integrate the Australian Curriculum content and achievement standards and any additional information, guidelines and rules to satisfy local requirements including advice on entry and exit points and credit for completed study.

The senior secondary Australian Curriculum for each subject should not, therefore, be read as a course of study. Rather, it is presented as content and achievement standards for integration into state and territory courses.

Senior secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics, with each subject organized into four units. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically, to make informed decisions. It provide students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.

General Mathematics focuses on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. It provides an opportunity to analyse and solve a wide range of geometrical problems in areas such as measurement, scaling, triangulation and navigation. It also provides opportunities to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series.

Mathematical Methods focuses on the development of the use of calculus and statistical analysis. The study of calculus in Mathematical Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Mathematical Methods develops the ability to describe and analyse phenomena involving uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. Specialist Mathematics is the only mathematics subject that cannot be taken as a stand-alone subject.

Structure of Mathematical Methods

Mathematical Methods is organised into four units. The topics broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. The probability and statistics topics lead to an introduction to statistical inference.

Unit 1	Unit 2	Unit 3	Unit 4
Functions and graphs Trigonometric functions Counting and probability	Exponential functions Arithmetic and geometric sequences and series Introduction to differential calculus	Further differentiation and applications Integrals Discrete random variables	The logarithmic function Continuous random variables and the normal distribution Interval estimates for proportions

Units

Unit 1 begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of functions and calculus. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of probability and statistics begins in this unit with a review of the fundamentals of probability, and the introduction of the concepts of conditional probability and independence. The study of the trigonometric functions begins with a consideration of the unit circle using degrees and the trigonometry of triangles and its application. Radian measure is introduced, and the graphs of the trigonometric functions are examined and their applications in a wide range of settings are explored.

In Unit 2, exponential functions are introduced and their properties and graphs examined. Arithmetic and geometric sequences and their applications are introduced and their recursive definitions applied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically (by calculating difference quotients), geometrically (as slopes of chords and tangents), and algebraically. This first calculus topic concludes with derivatives of polynomial functions, using simple applications of the derivative to sketch curves, calculate slopes and equations of tangents, determine instantaneous velocities, and solve optimisation problems.

In Unit 3, the study of calculus continues by introducing the derivatives of exponential and trigonometric functions and their applications, as well as some basic differentiation techniques and the concept of a second derivative, its meaning and applications. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The unit includes integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. Discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference.

In Unit 4, the logarithmic function and its derivative are studied. Continuous random variables are introduced and their applications examined. Probabilities associated with continuous distributions are calculated using definite integrals. In this unit students are introduced to one of the most important parts of statistics, namely statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit, inference is restricted to estimating proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained *a priori* without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

- Given a specified context (as described in the curriculum content)
- With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
- Students perform a specified action (described through a verb)
- In relation to what is valued in the curriculum (specified as the object or subject)
- With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as ‘analyse’ and ‘describe’ have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

Links to Foundation to Year 10

In Mathematical Methods, there is a strong emphasis on mutually reinforcing proficiencies in Understanding, Fluency, Problem solving and Reasoning. Students gain fluency in a variety of mathematical and statistical skills, including algebraic manipulations, constructing and interpreting graphs, calculating derivatives and integrals, applying probabilistic models, estimating probabilities and parameters from data, and using appropriate technologies. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving. In order to study Mathematical Methods, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended for the study of Mathematical Methods:

- ACMNA264: Define rational and irrational numbers, and perform operations with surds and fractional indices
- ACMNA269: Factorise monic and non-monic quadratic expressions, and solve a wide range of quadratic equations derived from a variety of contexts
- ACMSP278: Calculate and interpret the mean and standard deviation of data, and use these to compare datasets.

Representation of General capabilities

The seven general capabilities of *Literacy*, *Numeracy*, *Information and Communication technology (ICT) capability*, *Critical and creative thinking*, *Personal and social capability, Ethical understanding,* and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains financial applications of Mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age. Students will also learn about the probability of certain events occurring and will therefore be well equipped to make informed decisions.

ICT in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical undertanding in Mathematics

In the senior years students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical behaviour. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Representation of Cross-curriculum priorities

The senior secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples histories and cultures.

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia’s place in the region. Through analysis of relevant data, students are provided with opportunities to further develop an understanding of the diverse nature of Asia’s environments and traditional and contemporary cultures.

Each of the senior Mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving.Therefore, teachers are encouraged to select contexts for discussion connected with sustainability. Through analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 1

Unit Description

This unit begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of calculus. The basic trigonometric functions are then introduced. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of inferential statistics begins in this unit with a review of the fundamentals of probability and the introduction of the concepts of conditional probability and independence. Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in algebra, functions, graphs, trigonometric functions and probability
- solve problems using algebra, functions, graphs, trigonometric functions and probability
- apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions and probability
- interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
- communicate their arguments and strategies when solving problems.

Content Descriptions

Topic 1: Functions and graphs

Lines and linear relationships:

- determine the coordinates of the midpoint of two points (ACMMM001)
- examine examples of direct proportion and linearly related variables (ACMMM002)
- recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient (ACMMM003)
- find the equation of a straight line given sufficient information; parallel and perpendicular lines (ACMMM004)
- solve linear equations. (ACMMM005)

Review of quadratic relationships:

- examine examples of quadratically related variables (ACMMM006)
- recognise features of the graphs of $y = x^2$, $y = a(x-b)^2 + c$, and $y = a(x-b)(x-c)$, including their parabolic nature, turning points, axes of symmetry and intercepts (ACMMM007)
- solve quadratic equations using the quadratic formula and by completing the square (ACMMM008)
- find the equation of a quadratic given sufficient information (ACMMM009)
- find turning points and zeros of quadratics and understand the role of the discriminant (ACMMM010)
- recognise features of the graph of the general quadratic $y = ax^2 + bx + c$. (ACMMM011)

Inverse proportion:

- examine examples of inverse proportion (ACMMM012)
- recognise features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$, including their hyperbolic shapes, and their asymptotes. (ACMMM013)

Powers and polynomials:

- recognise features of the graphs of $y = x^n$ for $n \in \boldsymbol{N},\ \ n = -1 \text{ and } n = ½$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM014)
- identify the coefficients and the degree of a polynomial (ACMMM015)
- expand quadratic and cubic polynomials from factors (ACMMM016)
- recognise features of the graphs of $y = x^3$, $y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM017)
- factorise cubic polynomials in cases where a linear factor is easily obtained (ACMMM018)
- solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained. (ACMMM019)

Graphs of relations:

- recognise features of the graphs of $x^2 + y^2 = r^2$ and $(x-a)^2 + (y-b)^2 = r^2$, including their circular shapes, their centres and their radii (ACMMM020)
- recognise features of the graph of $y^2 = x$ including its parabolic shape and its axis of symmetry. (ACMMM021)

Functions:

- understand the concept of a function as a mapping between sets, and as a rule or a formula that defines one variable quantity in terms of another (ACMMM022)
- use function notation, domain and range, independent and dependent variables (ACMMM023)
- understand the concept of the graph of a function (ACMMM024)
- examine translations and the graphs of $y = f(x) + a$ and $y = f(x + b)$ (ACMMM025)
- examine dilations and the graphs of $y = cf(x)$ and $y = f(kx)$ (ACMMM026)
- recognise the distinction between functions and relations, and the vertical line test. (ACMMM027)

Topic 2: Trigonometric functions

Cosine and sine rules:

- review sine, cosine and tangent as ratios of side lengths in right-angled triangles (ACMMM028)
- understand the unit circle definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ and periodicity using degrees (ACMMM029)
- examine the relationship between the angle of inclination of a line and the gradient of that line (ACMMM030)
- establish and use the sine and cosine rules and the formula $Area = \frac{1}{2}bc\sin A$for the area of a triangle. (ACMMM031)

Circular measure and radian measure:

- define and use radian measure and understand its relationship with degree measure (ACMMM032)
- calculate lengths of arcs and areas of sectors in circles. (ACMMM033)

Trigonometric functions:

- understand the unit circle definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ and periodicity using radians (ACMMM034)
- recognise the exact values of $\sin\theta, \cos\theta \text{ and } \tan\theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (ACMMM035)
- recognise the graphs of $y = \sin x,\ y = \cos x, \text{ and } y = \tan x$ on extended domains (ACMMM036)
- examine amplitude changes and the graphs of $y = a\sin x$ and $y = a\cos x$ (ACMMM037)
- examine period changes and the graphs of $y = \sin bx,\ \ y = \cos bx$, and $y = \tan\ bx$ (ACMMM038)
- examine phase changes and the graphs of $y = \sin(x + c),\ \ y = \cos(x + c)$ and (ACMMM039)
- $y = \tan\ (x + c)$ and the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin x$ (ACMMM040)
- prove and apply the angle sum and difference identities (ACMMM041)
- identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems (ACMMM042)
- solve equations involving trigonometric functions using technology, and algebraically in simple cases. (ACMMM043)

Topic 3: Counting and probability

Combinations:

- understand the notion of a combination as an unordered set of r objects taken from a set of n distinct objects (ACMMM044)
- use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects (ACMMM045)

- expand $(x+y)^n$ for small positive integers n (ACMMM046)
- recognise the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $(x+y)^n$) (ACMMM047)
- use Pascal's triangle and its properties. (ACMMM048)

Language of events and sets:

- review the concepts and language of outcomes, sample spaces and events as sets of outcomes (ACMMM049)
- use set language and notation for events, including $\overline{A}$ (or A') for the complement of an event A, $A?B$ for the intersection of events A and B, and $A?B$ for the union, and recognise mutually exclusive events (ACMMM050)
- use everyday occurrences to illustrate set descriptions and representations of events, and set operations. (ACMMM051)

Review of the fundamentals of probability:

- review probability as a measure of 'the likelihood of occurrence' of an event (ACMMM052)
- review the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A)=0$ if A is an impossibility and $P(A)=1$ if A is a certainty (ACMMM053)
- review the rules: $P\left(\overline{A}\right)=1-P(A)$ and $P(A \cup B)=P(A)+P(B)-P(A \cap B)$ (ACMMM054)
- use relative frequencies obtained from data as point estimates of probabilities. (ACMMM055)

Conditional probability and independence:

- understand the notion of a conditional probability and recognise and use language that indicates conditionality (ACMMM056)
- use the notation $P(A|B)$ and the formula $P(A \cap B)=P(A|B)P(B)$ (ACMMM057)
- understand the notion of independence of an event A from an event B, as defined by $P(A|B)=P(A)$ (ACMMM058)
- establish and use the formula $P(A \cap B)=P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence (ACMMM059)
- use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events. (ACMMM060)

Unit 2

Unit Description

The algebra section of this unit focuses on exponentials and logarithms. Their graphs are examined and their applications in a wide range of settings are explored. Arithmetic and geometric sequences are introduced and their applications are studied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an ‘instantaneous rate of change’. These concepts are reinforced numerically, by calculating difference quotients both geometrically, as slopes of chords and tangents, and algebraically. Calculus is developed to study the derivatives of polynomial functions, with simple applications of the derivative to curve sketching, calculating slopes and equations of tangents, determining instantaneous velocities and solving optimisation problems.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques used in algebra, sequences and series, functions, graphs and calculus
- solve problems in algebra, sequences and series, functions, graphs and calculus
- apply reasoning skills in algebra, sequences and series, functions, graphs and calculus
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems
- communicate arguments and strategies when solving problems.

Content Descriptions

Topic 1: Exponential functions

Indices and the index laws:

- review indices (including fractional indices) and the index laws (ACMMM061)
- use radicals and convert to and from fractional indices (ACMMM062)
- understand and use scientific notation and significant figures. (ACMMM063)

Exponential functions:

- establish and use the algebraic properties of exponential functions (ACMMM064)
- recognise the qualitative features of the graph of $y = a^x \ (a > 0)$ including asymptotes, and of its translations ($y = a^x + b$ and $y = a^{x+c}$) (ACMMM065)
- identify contexts suitable for modelling by exponential functions and use them to solve practical problems (ACMMM066)
- solve equations involving exponential functions using technology, and algebraically in simple cases. (ACMMM067)

Topic 2 Arithmetic and geometric sequences and series

Arithmetic sequences:

- recognise and use the recursive definition of an arithmetic sequence: $t_{n+1} = t_n + d$ (ACMMM068)
- use the formula $t_n = t_1 + (n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature (ACMMM069)
- use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest (ACMMM070)
- establish and use the formula for the sum of the first n terms of an arithmetic sequence. (ACMMM071)

Geometric sequences:

- recognise and use the recursive definition of a geometric sequence: $t_{n+1} = rt_n$ (ACMMM072)
- use the formula $t_n = r^{n-1}t_1$ for the general term of a geometric sequence and recognise its exponential nature (ACMMM073)
- understand the limiting behaviour as $n \to \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r (ACMMM074)
- establish and use the formula $S_n = t_1 \frac{r^n - 1}{r - 1}$ for the sum of the first n terms of a geometric sequence (ACMMM075)
- use geometric sequences in contexts involving geometric growth or decay, such as compound interest. (ACMMM076)

Topic 3: Introduction to differential calculus

Rates of change:

- interpret the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of a function f (ACMMM077)
- use the Leibniz notation δx and δy for changes or increments in the variables x and y (ACMMM078)
- use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y = f(x)$ (ACMMM079)

- interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y = f(x)$. (ACMMM080)

The concept of the derivative:

- examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit (ACMMM081)
- define the derivative $f'(x)$ as $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ (ACMMM082)
- use the Leibniz notation for the derivative: $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx} = f'(x)$ where $y = f(x)$ (ACMMM083)
- interpret the derivative as the instantaneous rate of change (ACMMM084)
- interpret the derivative as the slope or gradient of a tangent line of the graph of $y = f(x)$. (ACMMM085)

Computation of derivatives:

- estimate numerically the value of a derivative, for simple power functions (ACMMM086)
- examine examples of variable rates of change of non-linear functions (ACMMM087)
- establish the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for positive integers n by expanding $(x+h)^n$ or by factorising $(x+h)^n - x^n$. (ACMMM088)

Properties of derivatives:

- understand the concept of the derivative as a function (ACMMM089)
- recognise and use linearity properties of the derivative (ACMMM090)
- calculate derivatives of polynomials and other linear combinations of power functions. (ACMMM091)

Applications of derivatives:

- find instantaneous rates of change (ACMMM092)
- find the slope of a tangent and the equation of the tangent (ACMMM093)
- construct and interpret position-time graphs, with velocity as the slope of the tangent (ACMMM094)
- sketch curves associated with simple polynomials; find stationary points, and local and global maxima and minima; and examine behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM095)
- solve optimisation problems arising in a variety of contexts involving simple polynomials on finite interval domains. (ACMMM096)

Anti-derivatives:

- calculate anti-derivatives of polynomial functions and apply to solving simple problems involving motion in a straight line. (ACMMM097)

Units 1 and 2 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of functions, calculus and statistics in routine and non-routine problems in a variety of contexts • selects and applies techniques in functions, calculus and statistics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models in routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of functions, calculus and statistics in routine and non-routine problems • selects and applies techniques in functions, calculus and statistics to solve routine and non-routine problems • selects and applies mathematical and statistical models in routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of functions, calculus and statistics that apply to routine problems • selects and applies techniques in functions, calculus and statistics to solve routine problems • applies mathematical and statistical models in routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of simple functions, calculus and statistics • uses simple techniques in functions, calculus and statistics in routine problems • demonstrates familiarity mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	• demonstrates limited familiarity with concepts of simple functions, calculus and statistics • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical or statistical models • uses digital technologies for arithmetic calculations and to display limited mathematical and statistical information

Reasoning and Communication

A	B	C	D	E
• represents functions, calculus and statistics in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments, which are succinct and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents functions, calculus and statistics in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments, which are clear and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains the limitations of models used when developing solutions to routine problems	• represents functions, calculus and statistics in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems • identifies the limitations of models used when developing solutions to routine problems	• represents simple functions and distributions in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the result of calculations • identifies the limitations of simple models used	• represents limited mathematical or statistical information in a structured context • communicates simple mathematical and statistical information • identifies solutions to routine problems • describes with limited familiarity the appropriateness of the results of calculations • identifies simple models

Unit 3

Unit Description

In this unit the study of calculus continues with the derivatives of exponential and trigonometric functions and their applications, together with some differentiation techniques and applications to optimisation problems and graph sketching. It concludes with integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. In statistics, discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. This supports the development of a framework for statistical inference.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Content Descriptions

Topic 1: Further differentiation and applications

Exponential functions:

- estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$ using technology, for various values of $a\ > 0$ (ACMMM098)
- recognise that e is the unique number a for which the above limit is 1 (ACMMM099)
- establish and use the formula $\frac{d}{dx}\left(e^x\right) = e^x$ (ACMMM100)
- use exponential functions and their derivatives to solve practical problems. (ACMMM101)

Trigonometric functions:

- establish the formulas $\frac{d}{dx}(\sin x) = \cos x, \quad \text{and} \quad \frac{d}{dx}(\cos x) = -\sin x$ by numerical estimations of the limits and informal proofs based on geometric constructions (ACMMM102)
- use trigonometric functions and their derivatives to solve practical problems. (ACMMM103)

Differentiation rules:

- understand and use the product and quotient rules (ACMMM104)
- understand the notion of composition of functions and use the chain rule for determining the derivatives of composite functions (ACMMM105)
- apply the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x,\ \frac{1}{x^n}$, $x \sin x,\ e^{-x} \sin x$ and $f(ax+b)$. (ACMMM106)

The second derivative and applications of differentiation:

- use the increments formula: $\delta y \cong \frac{dy}{dx} \times \delta x$ to estimate the change in the dependent variable y resulting from changes in the independent variable x (ACMMM107)
- understand the concept of the second derivative as the rate of change of the first derivative function (ACMMM108)
- recognise acceleration as the second derivative of position with respect to time (ACMMM109)
- understand the concepts of concavity and points of inflection and their relationship with the second derivative (ACMMM110)
- understand and use the second derivative test for finding local maxima and minima (ACMMM111)
- sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection (ACMMM112)
- solve optimisation problems from a wide variety of fields using first and second derivatives. (ACMMM113)

Topic 2: Integrals

Anti-differentiation:

- recognise anti-differentiation as the reverse of differentiation (ACMMM114)
- use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals (ACMMM115)
- establish and use the formula $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$ for $n \neq -1$ (ACMMM116)
- establish and use the formula $\int e^x dx = e^x + c$ (ACMMM117)

- establish and use the formulas $\int \sin x dx = -\cos x + c$ and $\int \cos x dx = \sin x + c$ (ACMMM118)
- recognise and use linearity of anti-differentiation (ACMMM119)
- determine indefinite integrals of the form $\int f(ax+b)dx$ (ACMMM120)
- identify families of curves with the same derivative function (ACMMM121)
- determine $f(x)$, given $f'\left(x\right)$ and an initial condition $f(a) = b$ (ACMMM122)
- determine displacement given velocity in linear motion problems. (ACMMM123)

Definite integrals:

- examine the area problem, and use sums of the form $\sum_i f(x_i)\ \delta x_i$ to estimate the area under the curve $y = f(x)$ (ACMMM124)
- interpret the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$ (ACMMM125)
- recognise the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\ \delta x_i$ (ACMMM126)
- interpret $\int_a^b f(x)dx$ as a sum of signed areas (ACMMM127)
- recognise and use the additivity and linearity of definite integrals. (ACMMM128)

Fundamental theorem:

- understand the concept of the signed area function $F(x) = \int_a^x f(t)dt$ (ACMMM129)
- understand and use the theorem: $F'(x) = \frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, and illustrate its proof geometrically (ACMMM130)
- understand the formula $\int_a^b f(x)dx = F(b) - F(a)$ and use it to calculate definite integrals. (ACMMM131)

Applications of integration:

- calculate the area under a curve (ACMMM132)
- calculate total change by integrating instantaneous or marginal rate of change (ACMMM133)
- calculate the area between curves in simple cases (ACMMM134)
- determine positions given acceleration and initial values of position and velocity. (ACMMM135)

Topic 3: Discrete random variables

General discrete random variables:

- understand the concepts of a discrete random variable and its associated probability function, and their use in modelling data (ACMMM136)
- use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable (ACMMM137)
- recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes (ACMMM138)
- examine simple examples of non-uniform discrete random variables (ACMMM139)
- recognise the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases (ACMMM140)
- recognise the variance and standard deviation of a discrete random variable as a measures of spread, and evaluate them in simple cases (ACMMM141)

- use discrete random variables and associated probabilities to solve practical problems. (ACMMM142)

Bernoulli distributions:

- use a Bernoulli random variable as a model for two-outcome situations (ACMMM143)
- identify contexts suitable for modelling by Bernoulli random variables (ACMMM144)
- recognise the mean p and variance $p(1-p)$ of the Bernoulli distribution with parameter p (ACMMM145)
- use Bernoulli random variables and associated probabilities to model data and solve practical problems. (ACMMM146)

Binomial distributions:

- understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial (ACMMM147)
- identify contexts suitable for modelling by binomial random variables (ACMMM148)
- determine and use the probabilities $\mathrm{P(X = r)} = \binom{\mathrm{n}}{\mathrm{r}}\mathrm{p^r(1-p)^{n-r}}$ associated with the binomial distribution with parameters n and p ; note the mean np and variance $\mathrm{np(1-p)}$ of a binomial distribution (ACMMM149)
- use binomial distributions and associated probabilities to solve practical problems. (ACMMM150)

Unit 4

Unit Description

The calculus in this unit deals with derivatives of logarithmic functions. In probability and statistics, continuous random variables and their applications are introduced and the normal distribution is used in a variety of contexts. The study of statistical inference in this unit is the culmination of earlier work on probability and random variables. Statistical inference is one of the most important parts of statistics, in which the goal is to estimate an unknown parameter associated with a population using a sample of data drawn from that population. In Mathematical Methods statistical inference is restricted to estimating proportions in two-outcome populations.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probabilty and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Content Descriptions

Topic 1: The logarithmic function

Logarithmic functions:

- define logarithms as indices: $a^x = b$ is equivalent to $x = \log_a b$ i.e. $a^{\log_a b} = b$ (ACMMM151)
- establish and use the algebraic properties of logarithms (ACMMM152)
- recognise the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a y$ (ACMMM153)
- interpret and use logarithmic scales such as decibels in acoustics, the Richter Scale for earthquake magnitude, octaves in music, pH in chemistry (ACMMM154)
- solve equations involving indices using logarithms (ACMMM155)
- recognise the qualitative features of the graph of $y = \log_a x$ $(a > 1)$ including asymptotes, and of its translations $y = \log_a x + b$ and $y = \log_a (x + c)$ (ACMMM156)
- solve simple equations involving logarithmic functions algebraically and graphically (ACMMM157)
- identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems. (ACMMM158)

Calculus of logarithmic functions:

- define the natural logarithm $\ln x = \log_e x$ (ACMMM159)
- recognise and use the inverse relationship of the functions $y = e^x$ and $y = \ln x$ (ACMMM160)
- establish and use the formula $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (ACMMM161)
- establish and use the formula $\int \frac{1}{x}\,dx = \ln\ x\ + c$, for $x > 0$ (ACMMM162)
- use logarithmic functions and their derivatives to solve practical problems. (ACMMM163)

Topic 2: Continuous random variables and the normal distribution

General continuous random variables:

- use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable (ACMMM164)
- understand the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts (ACMMM165)
- recognise the expected value, variance and standard deviation of a continuous random variable and evaluate them in simple cases (ACMMM166)
- understand the effects of linear changes of scale and origin on the mean and the standard deviation. (ACMMM167)

Normal distributions:

- identify contexts such as naturally occurring variation that are suitable for modelling by normal random variables (ACMMM168)
- recognise features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution (ACMMM169)
- calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve

practical problems. (ACMMM170)

Topic 3: Interval estimates for proportions

Random sampling:

- understand the concept of a random sample (ACMMM171)
- discuss sources of bias in samples, and procedures to ensure randomness (ACMMM172)
- use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli. (ACMMM173)

Sample proportions:

- understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{(p(1-p)/n}$ of the sample proportion $\hat{p}$ (ACMMM174)
- examine the approximate normality of the distribution of $\hat{p}$ for large samples (ACMMM175)
- simulate repeated random sampling, for a variety of values of *p* and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p} - p}{\sqrt{(\hat{p}(1-\hat{p})/n}}$ where the closeness of the approximation depends on both *n* and *p.* (ACMMM176)

Confidence intervals for proportions:

- the concept of an interval estimate for a parameter associated with a random variable (ACMMM177)
- use the approximate confidence interval $\left(\hat{p} - z\sqrt{(\hat{p}(1-\hat{p})/n}, \ \hat{p} + z\sqrt{(\hat{p}(1-\hat{p})/n}\right)$, as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution (ACMMM178)
- define the approximate margin of error $E = z\sqrt{(\hat{p}(1-\hat{p})/n}$ and understand the trade-off between margin of error and level of confidence (ACMMM179)
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain p. (ACMMM180)

Units 3 and 4 Achievement Standards

Concepts and techniques

A	B	C	D	E
• demonstrates knowledge of concepts of functions, integration and distributions in routine and non-routine problems in a variety of contexts • selects and applies techniques in functions, integration and distributions to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models in routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of functions, integration and distributions in routine and non-routine problems • selects and applies techniques in functions, integration and distributions to solve routine and non-routine problems • selects and applies mathematical and statistical models in routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of functions, integration and distributions that apply to routine problems • selects and applies techniques in functions, integration and distributions to solve routine problems • applies mathematical and statistical models in routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of simple functions, integration and distributions • uses simple techniques in functions, integration and distributions in routine problems • demonstrates familiarity with mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	• demonstrates limited familiarity with concepts of simple functions, integration and distributions • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical or statistical models • uses digital technologies for arithmetic calculations and to display limited mathematical and statistical information

Reasoning and Communication

A	B	C	D	E
• represents functions, integration and distributions in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments, which are succinct and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents functions, integration and distributions in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments, which are clear and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains the limitations of models used when developing solutions to routine problems	• represents functions, integration and distributions in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems • identifies thelimitations of models used when developing solutions to routine problems	• represents simple functions and distributions in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the result of calculations • identifies limitations of simple models used	• represents limited mathematical or statistical information in a structured context • communicates simple mathematical and statistical information • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

Mathematical Methods Glossary

Additivity property of definite integrals

The additivity property of definite integrals refers to ‘addition of intervals of integration’:

$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ for any numbers a, b and c and any function $f(x)$.

Algebraic properties of exponential functions

The algebraic properties of exponential functions are the index laws: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, for any real numbers $x,\ y,\ \text{and}\ a$, with $a > 0$

Algebraic properties of logarithms

The algebraic properties of logarithms are the rules: $\log_a(xy) = \log_a x + \log_a y$,$\log_a \frac{1}{x} = -\log_a x$, and $\log_a 1 = 0$, for any positive real numbers $x,\ y\ \text{and}\ a$

Antidifferentiation

An anti-derivative, primitive or indefinite integral of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'\left(x\right) = f\left(x\right)$.

The process of solving for anti-derivatives is called anti-differentiation.

Anti-derivatives are not unique. If $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number. We write $\int f(x)dx = F(x) + c$ to denote the set of all anti-derivatives of $f(x)$. The number c is called the constant of integration. For example, since $\frac{d}{dx}\left(x^3\right) = 3x^2$, we can write $\int 3x^2\, dx = x^3 + c$

Arithmetic sequence

An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence

2, 5, 8, 11, 14, 17, …

is an arithmetic sequence with common difference 3.

If the initial term of an arithmetic sequence is a and the common difference of successive members is d, then the nth term tn, of the sequence, is given by:

$t_n = a + (n - 1)d$ for $n = 1$

A recursive definition is

$t_1 = a$, $t_{n+1} = t_{n + d}$ where d is the common difference and n = 1.

Asymptote

A line is an asymptote to a curve if the distance between the line and the curve approaches zero as they ‘tend to infinity’. For example, the line with equation $x = \pi/2$ is a vertical asymptote to the graph of $y = \tan x$, and the line with equation $y = 0$ is a horizontal asymptote to the graph of $y = 1/x$.

Bernoulli random variable

A Bernoulli random variable has two possible values, namely 0 and 1. The parameter associated with such a random variable is the probability p of obtaining a 1.

Bernoulli trial

A Bernoulli trial is a chance experiment with possible outcomes, typically labeled 'success' and failure'.

Binomial distribution

The expansion $(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + y^n$ is known as the binomial theorem. The numbers $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n\times(n-1)\times\cdots\times(n-r+1)}{r\times(r-1)\times\cdots\times2\times1}$ are called binomial coefficients.

Central limit theorem

There are various forms of the Central limit theorem, a result of fundamental importance in statistics. For the purposes of this course, it can be expressed as follows:

"If X is the mean of n independent values of random variable X which has a finite mean μ and a finite standard deviation σ, then as $n \to \infty$ the distribution of $\frac{X-\mu}{\sigma/\sqrt{n}}$ approaches the standard normal distribution."

In the special case where X is a Bernoulli random variable with parameter p,X is the sample proportion $\hat{p}$,$\mu = p$ and $\sigma = \sqrt{p(1-p)}$. In this case the Central limit theorem is a statement that as $n \to \infty$ the distribution of $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$ approaches the standard normal distribution.

Chain rule

The chain rule relates the derivative of the composite of two functions to the functions and their derivatives.

If $h(x) = f \circ g(x)$ then $(f \circ g)^{'}(x) = f^{'}(g(x))g^{'}(x)$,

and in Leibniz notation: $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$

Circular measure

is the measurement of angle size in radians.

Completing the square

The quadratic expression $ax^2 + bx + c$ can be rewritten as $a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$. Rewriting it in this way is called completing the square.

Composition of functions

If $\mathrm{y} = g(x)$ and $z = f(y)$ for functions f and g, then z is a composite function of x. We write $z = f \circ g(x) = f(g(x))$. For example, $z = \sqrt{x^2+3}$ expresses z as a composite of the functions $\mathrm{f}(\mathrm{y}) = \sqrt{\mathrm{y}}$ and $\mathrm{g}(\mathrm{x}) = \mathrm{x}^2 + 3$

Concave up and concave down

A graph of $y = f(x)$ is concave up at a point P if points on the graph near P lie above the tangent at P. The graph is concave down at P if points on the graph near P lie below the tangent at P.

Conditional probability

The probability that an event A occurs can change if it becomes known that another event B occurs. The new probability is known as a conditional probability and is written as $P(A|B)$. If B has occurred, the sample space is reduced by discarding all outcomes that are not in the event B. The new sample space, called the reduced sample space, is B. The conditional probability of event A is given by $P\left(A\middle|B\right) = \frac{P(A\cap B)}{P(B)}$.

Discriminant

The discriminant of the quadratic expression $ax^2 + bx + c$ is the quantity $b^2 - 4ac$

Effect of linear change

The effects of linear changes of scale and origin on the mean and variance of a random variable are summarized as follows:

If X is a random variable and $Y = aX + b$, where a and b are constants, then

$E(Y) = aE(X) + b$ and $Var(Y) = a^2Var(X)$

Euler's number

Euler's number$e$ is an irrational number whose decimal expansion begins

$e = 2.7182818284590452353602874713527\cdots$

It is the base of the natural logarithms, and can be defined in various ways including:

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ and $e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$.

Expected value

The expected value$E(X)$ of a random variable X is a measure of the central tendency of its distribution.

If X is discrete, $E(X) = \sum_i p_i x_i$, where the x_i are the possible values of X and

$p_i = P(X = x_i)$.

If X is continuous, $E(x) = \int_{-\infty}^{\infty} xp(x)dx$, where $p(x)$ is the probability density function of X

Function

A function$f$ is a rule that associates with each element x in a set S a unique element $f(x)$ in a set T. We write $x \longmapsto f(x)$ to indicate the mapping of x to $f(x)$. The set S is called the domain of f and the set T is called the codomain. The subset of T consisting of all the elements $f(x) : x \in S$ is called the range of f. If we write $y = f(x)$ we say that x is the independent variable and y is the dependent variable.

Geometric sequence

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the common ratio. For example, the sequence

3, 6, 12, 24, ...

is a geometric sequence with common ratio 2. Similarly the sequence

40, 20, 10, 5, 2.5, …

is a geometric sequence with common ratio $\frac{1}{2}$.

If the initial term of a geometric sequence is a and the common ratio of successive members is r, then the nth term tn, of the sequence, is given by:

$t_n = ar^{n-1}$ for $n = 1$

A recursive definition is

$t_1 = a$, $t_{n+1} = rt_n$ for n = 1 and where r is the constant ratio

Gradient (Slope)

The gradient of the straight line passing through points (x_1, y_1) and (x_2, y_2) is the ratio $\frac{y_2-y_1}{x_2-x_1}$. Slope is a synonym for gradient.

Graph of a function

The graph of a function f is the set of all points (x, y) in Cartesian plane where x is in the domain of f and $y = f(x)$

Independent events

Two events are independent if knowing that one occurs tells us nothing about the other. The concept can be defined formally using probabilities in various ways: events A and B are independent if $P(A \cap B) = P(A)P(B)$, if $P(A|B) = P(A)$ or if $P(B) = P(B|A)$. For events A and B with non-zero probabilities, any one of these equations implies any other.

Index laws

The index laws are the rules: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, and $(ab)^x = a^x b^x$, for any real numbers $x,\ y,\ a \text{ and } b$, with $a > 0 \text{ and } b > 0$

Length of an arc

The length of an arc in a circle is given by $l = r\theta$, where l is the arc length, r is the radius and θ is the angle subtended at the centre, measured in radians. This is simply a rearrangement of the formula defining the radian measure of an angle.

Level of confidence

The level of confidence associated with a confidence interval for an unknown population parameter is the probability that a random confidence interval will contain the parameter.

linearity property of the derivative

The linearity property of the derivative is summarized by the equations:

$\frac{d}{dx}(ky) = k\frac{dy}{dx}$ for any constant k

and $\frac{d}{dx}(y_1 + y_2) = \frac{dy_1}{dx} + \frac{dy_2}{dx}$

Local and global maximum and minimum

A stationarypoint on the graph $y = f(x)$ of a differentiable function is a point where $f'(x) = 0$.

We say that $f(x_0)$ is a local maximum of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a global maximum of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x in the domain of f.

We say that $f(x_0)$ is a local minimum of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a global minimum of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x in the domain of f.

Margin of error

The margin of error of a confidence interval of the form $f - E < p < f + E$ is E, the half-width of the confidence interval. It is the maximum difference between f and p if p is actually in the confidence interval.

Mean of a random variable

The mean of a random variable is another name for its expected value.

identity matrix The variance$Var(X)$ of a random variable X is a measure of the ‘spread’ of its distribution.

If X is discrete, $Var(X) = \sum_i p_i (x_i - \mu)^2$, where $\mu = E(X)$ is the expected value.

If X is continuous, $Var(X) = \int_{-\infty}^{\infty} (x - \mu)^2 p(x) dx$

Mutually exclusive

Two events are mutually exclusive if there is no outcome in which both events occur.

Partial sum of an arithmetic sequence (Arithmetic series)

The partial sum Sn of the first n terms of an arithmetic sequence with first term a and common difference d.

a, *a* + *d*, *a* + 2*d*,……, *a* + (*n* – 1)*d*,…

is

Sn = $\frac{n}{2}$ (a + tn) = $\frac{n}{2}$ (2a + (n – 1)d) where tn is the nth term of the sequence.

The partial sums form a sequence with Sn+1 = Sn + tn and S1 = t1

Partial sums of a geometric sequence (Geometric series)

The partial sum Sn of the first n terms of a geometric sequence with first term a and common ratio r,

$a, ar, ar^2, \ldots\ldots, ar^{n-1}, \ldots$

is

$S_n = \frac{a(r^n - 1)}{r-1}$, $r \neq 1$.

The partial sums form a sequence with Sn+1 = Sn + tn and S1 = t1.

Partial sums of a sequence (Series)

The sequence of partial sums of a sequence t1,…, ts, … is defined by

$S_n = t_1 + \ldots + t_n$

Pascal's triangle

Pascal's triangle is a triangular arrangement of binomial coefficients. The n^{th} row consists of the binomial coefficients $\binom{n}{r}$, for $0 \leq r \leq n$, each interior entry is the sum of the two entries above it, and sum of the entries in the n^{th} row is 2^n

Period of a function

The period of a function $f(x)$ is the smallest positive number p with the property that $f(x+p) = f(x)$ for all x. The functions $\sin x$ and $\cos x$ both have period 2π and $\tan x$ has period π

Point and interval estimates

In statistics estimation is the use of information derived from a sample to produce an estimate of an unknown probability or population parameter. If the estimate is a single number, this number is called a point estimate. An interval estimate is an interval derived from the sample that, in some sense, is likely to contain the parameter.

A simple example of a point estimate of the probability p of an event is the relative frequency f of the event in a large number of Bernoulli trials. An example of an interval estimate for p is a confidence interval centred on the relative frequency f.

Point of inflection

A point P on the graph of $y = f(x)$ is a point of inflection if the concavity changes at P, i.e. points near P on one side of P lie above the tangent at P and points near P on the other side of P lie below the tangent at P

Probability density function

The probability density function of a continuous random variable is a function that describes the relative likelihood that the random variable takes a particular value. Formally, if $p(x)$ is the probability density of the continuous random variable X, then the probability that X takes a value in some interval $[a, b]$ is given by $\int_a^b p(x)\, dx$.

Probability distribution

The probability distribution of a discrete random variable is the set of probabilities for each of its possible values.

Product rule

The product rule relates the derivative of the product of two functions to the functions and their derivatives.

If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g\left(x\right)$,

and in Leibniz notation: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

Quadratic formula

If $ax^2 + bx + c = 0$ with $a \neq 0$, then $x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$. This formula for the roots is called the quadratic formula.

Quantile

A quantilet_α for a continuous random variable X is defined by $P(X > t_\alpha) = \alpha$, where $0 < \alpha < 1$.

The median$m$ of X is the quantile corresponding to $\alpha = 0.5$: $P(X > m) = 0.5$

Quotient rule

The quotient rule relates the derivative of the quotient of two functions to the functions and their derivatives

If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

and in Leibniz notation: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Radian measure

The radian measureθ of an angle in a sector of a circle is defined by , where is the radius and is the arc length. Thus an angle whose degree measure is has radian measure .

Random variable

A random variable is a numerical quantity whose value depends on the outcome of a chance experiment. Typical examples are the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin, and the number of tonnes of wheat produced in Australia in a year.

A discrete random variable is one whose possible values are the counting numbers , or form a finite set, as in the first two examples.

A continuous random variable is one whose set of possible values are all of the real numbers in some interval.

Relative frequency

If an event occurs times when a chance experiment is repeated times, the relative frequency of is .

Unit 2

Secant

A secant of the graph of a function is the straight line passing through two points on the graph. The line segment between the two points is called a chord.

Second derivative test

According to the second derivative test, if then is a local maximum of if and is a local minimum if

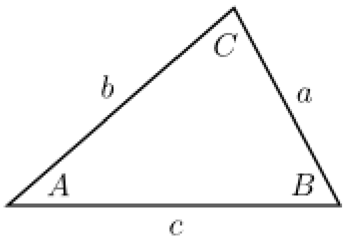
Sine and cosine functions

In the unit circle definition of cosine and sine, are the coordinates of the point on the unit circle corresponding to the angle

Sine rule and cosine rule

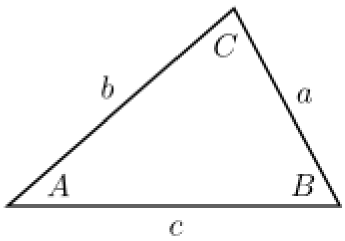
The lengths of the sides of a triangle are related to the sines of its angles by the equations

This is known as the sine rule.



The lengths of the sides of a triangle are related to the cosine of one of its angles by the equation

This is known as the cosine rule.



Standard deviation of a random variable

The standard deviation of a random variable is the square root of its variance.

Tangent line

The tangent line (or simply the tangent) to a curve at a given point can be described intuitively as the straight line that "just touches" the curve at that point. At the point where the tangent touches the curve, the curve has “the same

direction” as the tangent line.”. In this sense it is the best straight-line approximation to the curve at the point .

The fundamental theorem of calculus

The fundamental theorem of calculus relates differentiation and definite integrals. It has two forms:

The linearity property of anti-differentiation

The linearity property of anti-differentiation is summarized by the equations:

for any constant and

for any two functions

Similar equations describe the linearity property of definite integrals:

for any constant and

for any two functions

Triangular continuous random variable

A triangular continuous random variable is one whose probability density function has a graph with the shape of a triangle.

Uniform continuous random variable

A uniform continuous random variable is one whose probability density function has constant value on the range of possible values of . If the range of possible values is the interval then if and otherwise.

Uniform discrete random variable

A uniform discrete random variable is one whose possible values have equal probability of occurrence. If there are possible values, the probability of occurrence of any one of them is .

Vertical line test

A relation between two real variables and is a function and for some function if and only if each vertical line, i.e. each line parallel to the axis, intersects the graph of the relation in at most one point. This test to determine whether a relation is, in fact, a function is known as the vertical line test.

Typesetting math: 94%

The Australian Curriculum
Specialist Mathematics

Specialist Mathematics

Rationale and Aims

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Because both mathematics and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout Specialist Mathematics. There is also a sound logical basis to this subject, and in mastering the subject students will develop logical reasoning skills to a high level.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical and statistical models more extensively. Topics are developed systematically and lay the foundations for future studies in quantitative subjects in a coherent and structured fashion. Students of Specialist Mathematics will be able to appreciate the true nature of mathematics, its beauty and its functionality.

Specialist Mathematics has been designed to be taken in conjunction with Mathematical Methods. The subject contains topics in functions, calculus, probability and statistics that build on and deepen the ideas presented in Mathematical Methods and demonstrate their application in many areas. Vectors, complex numbers and matrices are introduced. Specialist Mathematics is designed for students with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at university.

For all content areas of Specialist Mathematics, the proficiency strands of the F–10 curriculum are still applicable and should be inherent in students' learning of the subject. These strands are Understanding, Fluency, Problem solving and Reasoning and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency of skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves, freeing up working memory for more complex aspects of problem solving. In Specialist Mathematics, the formal explanation of reasoning through mathematical proof takes on an important role and the ability to present the solution of any problem in a logical and clear manner is of paramount importance. The ability to transfer skills learned to solve one class of problems, for example integration, to solve another class of problems, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this subject.

Specialist Mathematics is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit blends algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. For example, in Unit 1 vectors for two-dimensional space are introduced and then in Unit 3 vectors are studied for three-dimensional space. The Unit 3 vector topic leads to the establishment of the equations of lines and planes and this in turn prepares students for an introduction to solving simultaneous equations in three variables. The study of calculus, which is developed in Mathematical Methods, is applied in Vectors in Unit 3 and applications of calculus and statistics in Unit 4.

Aims

Specialist Mathematics aims to develop students':

- understanding of concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- ability to solve applied problems using concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- capacity to choose and use technology appropriately.
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- ability to construct proofs.

Organisation

Overview of senior secondary Australian Curriculum

ACARA has developed senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications. The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

The senior secondary Australian Curriculum specifies content and achievement standards for each senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) expected of students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment and examinations). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years.

State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into their courses. They will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and Board. They will determine the assessment and certification specifications for their local courses that integrate the Australian Curriculum content and achievement standards and any additional information, guidelines and rules to satisfy local requirements including advice on entry and exit points and credit for completed study.

The senior secondary Australian Curriculum for each subject should not, therefore, be read as a course of study. Rather, it is presented as content and achievement standards for integration into state and territory courses.

Senior secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics, with each subject organised into four units. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.

General Mathematics focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve analysing univariate and bivariate data, including time series data.

Mathematical Methods focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. Specialist Mathematics is the only mathematics subject that cannot be taken as a stand-alone subject.

Structure of Specialist Mathematics

Specialist Mathematics is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit provides a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. For example, vectors in the plane are introduced in Unit 1 and then in Unit 3 they are studied for three-dimensional space. In Unit 3, the topic 'Vectors in three dimensions' leads to the establishment of the equations of lines and planes, and this in turn prepares students for solving simultaneous equations in three variables.

Unit 1	Unit 2	Unit 3	Unit 4
Combinatorics Vectors in the plane Geometry	Trigonometry Matrices Real and complex numbers	Complex numbers Functions and sketching graphs Vectors in three dimensions	Integration and applications of integration Rates of change and differential equations Statistical inference

Units

Unit 1 contains three topics that complement the content of Mathematical Methods. The proficiency strand, 'Reasoning', of the F–10 curriculum is continued explicitly in the topic 'Geometry' through a discussion of developing mathematical arguments. This topic also provides the opportunity to summarise and extend students' studies in Euclidean Geometry, knowledge which is of great benefit in the later study of topics such as vectors and complex numbers. The topic 'Combinatorics' provides techniques that are very useful in many areas of mathematics, including probability and algebra. The topic 'Vectors in the plane' provides new perspectives on working with two-dimensional space, and serves as an introduction to techniques which can be extended to three-dimensional space in Unit 3. These three topics considerably broaden students' mathematical experience and therefore begin an awakening to the breadth and utility of the subject. They also enable students to increase their mathematical flexibility and versatility.

Unit 2 contains three topics, 'Trigonometry', 'Matrices' and 'Real and complex numbers'. 'Matrices' provides new perspectives for working with two-dimensional space, 'Real and complex numbers' provides a continuation of the study of numbers. The topic 'Trigonometry' contains techniques that are used in other topics in both this unit and Units 3 and 4. All of these topics develop students' ability to construct mathematical arguments. The technique of proof by the principle of mathematical induction is introduced in this unit.

Unit 3 contains three topics, ‘Complex numbers’, ‘Vectors in three dimensions’, and ‘Functions and sketching graphs’. The Cartesian form of complex numbers was introduced in Unit 2, and in Unit 3 the study of complex numbers is extended to the polar form. The study of functions and techniques of calculus begun in Mathematical Methods is extended and utilised in the sketching of graphs and the solution of problems involving integration. The study of vectors begun in Unit 1, which focused on vectors in one- and two-dimensional space, is extended in Unit 3 to three-dimensional vectors, vector equations and vector calculus, with the latter building on students’ knowledge of calculus from Mathematical Methods. Cartesian and vector equations, together with equations of planes,enables students to solve geometric problems and to solve problems involving motion in three-dimensional space.

Unit 4 contains three topics: ‘Integration and applications of integration’, ‘Rates of change and differential equations’ and ‘Statistical inference’. In this unit, the study of differentiation and integration of functions is continued, and the techniques developed from this and previous topics in calculus are applied to the area of simple differential equations, in particular in biology and kinematics. These topics serve to demonstrate the applicability of the mathematics learnt throughout this course. Also in this unit, all of the students’ previous experience in statistics is drawn together in the study of the distribution of sample means. This is a topic that demonstrates the utility and power of statistics.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained *a priori* without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

- Given a specified context (as described in the curriculum content)
- With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
- Students perform a specified action (described through a verb)
- In relation to what is valued in the curriculum (specified as the object or subject)
- With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as ‘analyse’ and ‘describe’ have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

Links to Foundation to Year 10

For all content areas of Specialist Mathematics, the proficiency strands of the F–10 curriculum are still very much applicable and should be inherent in students’ learning of the subject. The strands of Understanding, Fluency, Problem solving and Reasoning are essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving.In Specialist Mathematics, the formal explanation of reasoning through mathematical proof takes an important role, and the ability to present the solution of any problem in a logical and clear manner is of paramount significance. The ability to transfer skills learned to solve one class of problems, such as integration, to solve another class of problems, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this subject. In order to study Specialist Mathematics, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended as preparation for Specialist Mathematics:

- ACMMG273: Establish the sine, cosine and area rules for any triangle, and solve related problems
- ACMMG274: Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies
- ACMNAP266: Investigate the concept of a polynomial, and apply the factor and remainder theorems to solve problems.

Representation of General capabilities

The seven general capabilities of *Literacy*, *Numeracy*, *Information and Communication technology (ICT) capability*, *Critical and creative thinking*, *Personal and social capability, Ethical understanding,* and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains topics that will equip students for the ever-increasing demands of the information age.

ICT in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skillfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical understanding in Mathematics

In the senior years students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical understanding. They develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person’s environment and language.

Representation of Cross-curriculum priorities

The senior secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander peoples' past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander peoples' histories and cultures.

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. By analysing relevant data, students have opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Each of the senior secondary mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 1

Unit Description

Unit 1 of Specialist Mathematics contains three topics – ‘Combinatorics’, ‘Vectors in the plane’ and ‘Geometry’ – that complement the content of Mathematical Methods. The proficiency strand, Reasoning, of the F–10 curriculum is continued explicitly in ‘Geometry’ through a discussion of developing mathematical arguments. While these ideas are illustrated through deductive Euclidean geometry in this topic, they recur throughout all of the topics in Specialist Mathematics. ‘Geometry’ also provides the opportunity to summarise and extend students’ studies in Euclidean Geometry. An understanding of this topic is of great benefit in the study of later topics in the course, including vectors and complex numbers.

‘Vectors in the plane’ provides new perspectives for working with two-dimensional space, and serves as an introduction to techniques that will be extended to three-dimensional space in Unit 3.

‘Combinatorics’ provides techniques that are useful in many areas of mathematics including probability and algebra. All these topics develop students’ ability to construct mathematical arguments.

These three topics considerably broaden students’ mathematical experience and therefore begin an awakening to the breadth and utility of the subject. They also enable students to increase their mathematical flexibility and versatility.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in combinatorics, geometry and vectors
- apply reasoning skills and solve problems in combinatorics, geometry and vectors
- communicate their arguments and strategies when solving problems
- construct proofs in a variety of contexts including algebraic and geometric
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Combinatorics

Permutations (ordered arrangements):

- solve problems involving permutations (ACMSM001)
- use the multiplication principle (ACMSM002)
- use factorial notation (ACMSM003)
- solve problems involving permutations and restrictions with or without repeated objects. (ACMSM004)

The inclusion-exclusion principle for the union of two sets and three sets:

- determine and use the formulas for finding the number of elements in the union of two and the union of three sets. (ACMSM005)

The pigeon-hole principle:

- solve problems and prove results using the pigeon-hole principle. (ACMSM006)

Combinations (unordered selections):

- solve problems involving combinations (ACMSM007)
- use the notation $\binom{n}{r}$ or ${}^{n}C_{r}$ (ACMSM008)
- derive and use simple identities associated with Pascal's triangle. (ACMSM009)

Topic 2: Vectors in the plane

Representing vectors in the plane by directed line segments:

- examine examples of vectors including displacement and velocity (ACMSM010)
- define and use the magnitude and direction of a vector (ACMSM011)
- represent a scalar multiple of a vector (ACMSM012)
- use the triangle rule to find the sum and difference of two vectors. (ACMSM013)

Algebra of vectors in the plane:

- use ordered pair notation and column vector notation to represent a vector (ACMSM014)
- define and use unit vectors and the perpendicular unit vectors *i* and *j* (ACMSM015)
- express a vector in component form using the unit vectors *i* and *j* (ACMSM016)
- examine and use addition and subtraction of vectors in component form (ACMSM017)
- define and use multiplication by a scalar of a vector in component form (ACMSM018)
- define and use scalar (dot) product (ACMSM019)
- apply the scalar product to vectors expressed in component form (ACMSM020)
- examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular (ACMSM021)
- define and use projections of vectors (ACMSM022)

- solve problems involving displacement, force and velocity involving the above concepts. (ACMSM023)

Topic 3: Geometry

The nature of proof:

- use implication, converse, equivalence, negation, contrapositive (ACMSM024)
- use proof by contradiction (ACMSM025)
- use the symbols for implication ($\Rightarrow$), equivalence ($\Longleftrightarrow$), and equality ($=$) (ACMSM026)
- use the quantifiers 'for all' and 'there exists' (ACMSM027)
- use examples and counter-examples. (ACMSM028)

Circle properties and their proofs including the following theorems:

- An angle in a semicircle is a right angle (ACMSM029)
- The angle at the centre subtended by an arc of a circle is twice the angle at the circumference subtended by the same arc (ACMSM030)
- Angles at the circumference of a circle subtended by the same arc are equal (ACMSM031)
- The opposite angles of a cyclic quadrilateral are supplementary (ACMSM032)
- Chords of equal length subtend equal angles at the centre and conversely chords subtending equal angles at the centre of a circle have the same length (ACMSM033)
- The alternate segment theorem (ACMSM034)
- When two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord (ACMSM035)
- When a secant (meeting the circle at A and B) and a tangent (meeting the circle at T) are drawn to a circle from an external point M, the square of the length of the tangent equals the product of the lengths to the circle on the secant. ($AM \times BM = TM^2$) (ACMSM036)
- Suitable converses of some of the above results (ACMSM037)
- Solve problems finding unknown angles and lengths and prove further results using the results listed above. (ACMSM038)

Geometric proofs using vectors in the plane including:

- The diagonals of a parallelogram meet at right angles if and only if it is a rhombus (ACMSM039)
- Midpoints of the sides of a quadrilateral join to form a parallelogram (ACMSM040)
- The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides. (ACMSM041)

Unit 2

Unit Description

Unit 2 of Specialist Mathematics contains three topics – ‘Trigonometry’, ‘Real and complex numbers’ and ‘Matrices’…

‘Trigonometry’ contains techniques that are used in other topics in both this unit and Unit 3. ‘Real and complex numbers’ provides a continuation of students’ study of numbers, and the study of complex numbers is continued in Unit 3. This topic also contains a section on proof by mathematical induction. The study of matrices is undertaken, including applications to linear transformations of the plane.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in trigonometry, real and complex numbers, and matrices
- apply reasoning skills and solve problems in trigonometry, real and complex numbers, and matrices
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Trigonometry

The basic trigonometric functions:

- find all solutions of $\mathrm{f}(\mathrm{a}(\mathrm{x}-\mathrm{b}))=\mathrm{c}$ where f is one of $\sin$, $\cos$ or $\tan$ (ACMSM042)
- graph functions with rules of the form $\mathrm{y}=\mathrm{f}(\mathrm{a}(\mathrm{x}-\mathrm{b}))$ where f is one of $\sin$, $\cos$ or $\tan$. (ACMSM043)

Compound angles:

- prove and apply the angle sum, difference and double angle identities. (ACMSM044)

The reciprocal trigonometric functions, secant, cosecant and cotangent:

- define the reciprocal trigonometric functions, sketch their graphs, and graph simple transformations of them. (ACMSM045)

Trigonometric identities:

- prove and apply the Pythagorean identities (ACMSM046)
- prove and apply the identities for products of sines and cosines expressed as sums and differences (ACMSM047)
- convert sums $\mathrm{a}\cos\mathrm{x}+\mathrm{b}\ \sin\mathrm{x}$ to $\mathrm{R}\ \cos(\mathrm{x}\pm\alpha)$ or $\mathrm{R}\sin(\mathrm{x}\pm\alpha)$ and apply these to sketch graphs, solve equations of the form $\mathrm{a}\cos\mathrm{x}+\mathrm{b}\sin\mathrm{x}=\mathrm{c}$ and solve problems (ACMSM048)
- prove and apply other trigonometric identities such as $\cos 3\mathrm{x}=4\cos^3\mathrm{x}-3\cos\mathrm{x}$. (ACMSM049)

Applications of trigonometric functions to model periodic phenomena:

- model periodic motion using sine and cosine functions and understand the relevance of the period and amplitude of these functions in the model. (ACMSM050)

Topic 2: Matrices

Matrix arithmetic:

- understand the matrix definition and notation (ACMSM051)
- define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and inverse (ACMSM052)
- calculate the determinant and inverse of 2×2 matrices and solve matrix equations of the form $\mathrm{AX}=\mathrm{B}$, where A is a 2×2 matrix and X and B are column vectors. (ACMSM053)

Transformations in the plane:

- translations and their representation as column vectors (ACMSM054)
- define and use basic linear transformations: dilations of the form $(\mathrm{x},\mathrm{y})\longrightarrow(\lambda_1\mathrm{x},\lambda_2\mathrm{y})$, rotations about the origin and reflection in a line which passes through the origin, and the representations of these transformations by 2×2 matrices (ACMSM055)
- apply these transformations to points in the plane and geometric objects (ACMSM056)
- define and use composition of linear transformations and the corresponding matrix products (ACMSM057)
- define and use inverses of linear transformations and the relationship with the matrix inverse (ACMSM058)
- examine the relationship between the determinant and the effect of a linear transformation on area (ACMSM059)

- establish geometric results by matrix multiplications; for example, show that the combined effect of two reflections in lines through the origin is a rotation. (ACMSM060)

Topic 3: Real and complex numbers

Proofs involving numbers:

- prove simple results involving numbers. (ACMSM061)

Rational and irrational numbers:

- express rational numbers as terminating or eventually recurring decimals and vice versa (ACMSM062)
- prove irrationality by contradiction for numbers such as $\sqrt{2}$ and $\log_2 5$. (ACMSM063)

An introduction to proof by mathematical induction:

- understand the nature of inductive proof including the 'initial statement' and inductive step (ACMSM064)
- prove results for sums, such as $1+4+9\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$ for any positive integer *n* (ACMSM065)
- prove divisibility results, such as $3^{2n+4}-2^{2n}$ is divisible by 5 for any positive integer *n.* (ACMSM066)

Complex numbers:

- define the imaginary number $\mathbf{i}$ as a root of the equation $\mathbf{x}^2=-1$ (ACMSM067)
- use complex numbers in the form $\mathbf{a}+\mathbf{bi}$ where $\mathbf{a}$ and $\mathbf{b}$ are the real and imaginary parts (ACMSM068)
- determine and use complex conjugates (ACMSM069)
- perform complex-number arithmetic: addition, subtraction, multiplication and division. (ACMSM070)

The complex plane:

- consider complex numbers as points in a plane with real and imaginary parts as Cartesian coordinates (ACMSM071)
- examine addition of complex numbers as vector addition in the complex plane (ACMSM072)
- understand and use location of complex conjugates in the complex plane. (ACMSM073)

Roots of equations:

- use the general solution of real quadratic equations (ACMSM074)
- determine complex conjugate solutions of real quadratic equations (ACMSM075)
- determine linear factors of real quadratic polynomials. (ACMSM076)

Units 1 and 2 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge and understanding of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers in routine and non-routine problems in a variety of contexts • synthesises information to select and apply techniques in mathematics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical models to routine and non-routine problems in a variety of contexts • constructs mathematical proofs in a variety of contexts, and adapts previously seen mathematical proofs • uses digital technologies effectively to graph, display and organise mathematical information to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers in routine and non-routine problems • selects and applies techniques in mathematics to solve routine and non-routine problems • selects and applies mathematical models to routine and non-routine problems • constructs simple mathematical proofs, and adapts previously seen mathematical proofs • uses digital technologies appropriately to graph, display and organise mathematical information to solve a range of routine and non-routine problems	• demonstrates knowledge of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers that apply to routine problems • selects and applies techniques in mathematics to solve routine problems • applies mathematical models to routine problems • reproduces previously seen mathematical proofs • uses digital technologies to graph, display and organise mathematical information to solve routine problems	• demonstrates knowledge of the concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers • uses simple techniques in mathematics in routine problems • demonstrates familiarity with mathematical models • reproduces previously seen simple mathematical proofs • uses digital technologies to display some mathematical information in routine problems	• demonstrates limited familiarity with simple concepts of vectors, combinatorics, geometry, matrices, trigonometry and complex numbers • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical models • demonstrates limited familiarity with mathematical proofs • uses digital technologies for arithmetic calculations and to display limited mathematical information

Reasoning and Communication

A	B	C	D	E
• represents mathematical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates succinct and reasoned mathematical judgments and arguments, including proofs, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents mathematical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear and reasoned mathematical judgments and arguments, including simple proofs, using appropriate language • interpret the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical information in numerical, graphical and symbolic form in routine problems • communicates mathematical arguments, including previously seen proofs, using appropriate language • interprets the solutions to routine problems • describes the reasonableness of the results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents mathematical information in numerical, graphical or symbolic form in routine problems • communicates mathematical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	• represents simple mathematical information in a structured context • communicates simple mathematical information • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

Unit 3

Unit Description

Unit 3 of Specialist Mathematics contains three topics: ‘Vectors in three dimensions’, ‘Complex numbers’ and ‘Functions and sketching graphs’. The study of vectors was introduced in Unit 1 with a focus on vectors in two-dimensional space. In this unit, three-dimensional vectors are studied and vector equations and vector calculus are introduced, with the latter extending students’ knowledge of calculus from Mathematical Methods. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space.The Cartesian form of complex numbers was introduced in Unit 2, and the study of complex numbers is now extended to the polar form.

The study of functions and techniques of graph sketching, begun in Mathematical Methods, is extended and applied in sketching graphs and solving problems involving integration.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students will:

- understand the concepts and techniques in vectors, complex numbers, functions and graph sketching
- apply reasoning skills and solve problems in vectors, complex numbers, functions and graph sketching
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Complex numbers

Cartesian forms:

- review real and imaginary parts $Re(z)$ and $Im(z)$ of a complex number z (ACMSM077)
- review Cartesian form (ACMSM078)
- review complex arithmetic using Cartesian forms. (ACMSM079)

Complex arithmetic using polar form:

- use the modulus $|z|$of a complex number z and the argument $Arg\ (z)$ of a non-zero complex number z and prove basic identities involving modulus and argument (ACMSM080)
- convert between Cartesian and polar form (ACMSM081)
- define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these (ACMSM082)
- prove and use De Moivre's theorem for integral powers. (ACMSM083)

The complex plane (the Argand plane):

- examine and use addition of complex numbers as vector addition in the complex plane (ACMSM084)
- examine and use multiplication as a linear transformation in the complex plane (ACMSM085)
- identify subsets of the complex plane determined by relations such as $|z-3i| \leq 4$ $\frac{\pi}{4} \leq Arg\left(z\right) \leq \frac{3\pi}{4}$, $Re(z) > Im(z)$ and $|z-1| = 2|z-i|$. (ACMSM086)

Roots of complex numbers:

- determine and examine the n^{th} roots of unity and their location on the unit circle (ACMSM087)
- determine and examine the n^{th}roots of complex numbers and their location in the complex plane. (ACMSM088)

Factorisation of polynomials:

- prove and apply the factor theorem and the remainder theorem for polynomials (ACMSM089)
- consider conjugate roots for polynomials with real coefficients (ACMSM090)
- solve simple polynomial equations. (ACMSM091)

Topic 2: Functions and sketching graphs

Functions:

- determine when the composition of two functions is defined (ACMSM092)
- find the composition of two functions (ACMSM093)
- determine if a function is one-to-one (ACMSM094)
- consider inverses of one-to-one function (ACMSM095)
- examine the reflection property of the graph of a function and the graph of its inverse. (ACMSM096)
- *Sketching graphs:* (ACMSM097)

- use and apply the notation $|x|$ for the absolute value for the real number x and the graph of $y = |x|$ (ACMSM098)
- examine the relationship between the graph of $y = f(x)$ and the graphs of $y = \frac{1}{f(x)}$,$y = |f(x)|$ and $y = f(|x|)$ (ACMSM099)
- sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree. (ACMSM100)

Topic 3: Vectors in three dimensions

The algebra of vectors in three dimensions:

- review the concepts of vectors from Unit 1 and extend to three dimensions including introducing the unit vectors i, j and k. (ACMSM101)
- prove geometric results in the plane and construct simple proofs in three-dimensions. (ACMSM102)

Vector and Cartesian equations:

- introduce Cartesian coordinates for three-dimensional space, including plotting points and the equations of spheres (ACMSM103)
- use vector equations of curves in two or three dimensions involving a parameter, and determine a 'corresponding' Cartesian equation in the two-dimensional case (ACMSM104)
- determine a vector equation of a straight line and straight-line segment, given the position of two points, or equivalent information, in both two and three dimensions (ACMSM105)
- examine the position of two particles each described as a vector function of time, and determine if their paths cross or if the particles meet (ACMSM106)
- use the cross product to determine a vector normal to a given plane (ACMSM107)
- determine vector and Cartesian equations of a plane and of regions in a plane. (ACMSM108)

Systems of linear equations:

- recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations (ACMSM109)
- examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables. (ACMSM110)

Vector calculus:

- consider position of vectors as a function of time (ACMSM111)
- derive the Cartesian equation of a path given as a vector equation in two dimensions including ellipses and hyperbolas (ACMSM112)
- differentiate and integrate a vector function with respect to time (ACMSM113)
- determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration (ACMSM114)
- apply vector calculus to motion in a plane including projectile and circular motion. (ACMSM115)

Unit 4

Unit Description

Unit 4 of Specialist Mathematics contains three topics: ‘Integration and applications of integration’, ‘Rates of change and differential equations’ and ‘Statistical inference’.

In Unit 4, the study of differentiation and integration of functions continues, and the calculus techniques developed in this and previous topics are applied to simple differential equations, in particular in biology and kinematics. These topics demonstrate the real-world applications of the mathematics learned throughout Specialist Mathematics.

In this unit all of the students’ previous experience working with probability and statistics is drawn together in the study of statistical inference for the distribution of sample means and confidence intervals for sample means.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in applications of calculus and statistical inference
- apply reasoning skills and solve problems in applications of calculus and statistical inference
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Topic 1: Integration and applications of integration

Integration techniques:

- integrate using the trigonometric identities $\sin^2 x = \frac{1}{2}\left(1 - \cos\, 2x\right)$, $\cos^2 x = \frac{1}{2}\left(1 + \cos\, 2x\right)$ and $1 +\ \tan^2 x = \sec^2 x$ (ACMSM116)
- use substitution *u* = *g*(*x*) to integrate expressions of the form $f(g(x))g^{'}(x)$ (ACMSM117)
- establish and use the formula $\int \frac{1}{x}\,dx = \ln\ |x\,| + c$, for $x \neq 0$ (ACMSM118)
- find and use the inverse trigonometric functions: arcsine, arccosine and arctangent (ACMSM119)
- find and use the derivative of the inverse trigonometric functions: arcsine, arccosine and arctangent (ACMSM120)
- integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$ (ACMSM121)
- use partial fractions where necessary for integration in simple cases (ACMSM122)
- integrate by parts. (ACMSM123)

Applications of integral calculus:

- calculate areas between curves determined by functions (ACMSM124)
- determine volumes of solids of revolution about either axis (ACMSM125)
- use numerical integration using technology (ACMSM126)
- use and apply the probability density function, $f(t) = \lambda e^{-\lambda t}$ for $t \geq 0$, of the exponential random variable with parameter $\lambda > 0$, and use the exponential random variables and associated probabilities and quantiles to model data and solve practical problems. (ACMSM127)

Topic 2: Rates of change and differential equations

- use implicit differentiation to determine the gradient of curves whose equations are given in implicit form (ACMSM128)
- Related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (ACMSM129)
- solve simple first-order differential equations of the form $\frac{dy}{dx} = f\left(x\right)$, differential equations of the form $\frac{dy}{dx} = g(y)$ and, in general, differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables (ACMSM130)
- examine slope (direction or gradient) fields of a first order differential equation (ACMSM131)
- formulate differential equations including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved. (ACMSM132)

Modelling motion:

- examine momentum, force, resultant force, action and reaction (ACMSM133)
- consider constant and non-constant force (ACMSM134)
- understand motion of a body under concurrent forces (ACMSM135)
- consider and solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and the use of expressions $\frac{dv}{dt}$, $v\,\frac{dv}{dx}$ and $\frac{d\left(\frac{1}{2}v^2\right)}{dx}$ for acceleration. (ACMSM136)

Topic 3: Statistical inference

Sample means:

- examine the concept of the sample mean X as a random variable whose value varies between samples where X is a random variable with mean μ and the standard deviation σ (ACMSM137)
- simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of $\overline{X}$ across samples of a fixed size n, including its mean μ, its standard deviation $\sigma/\sqrt{n}$ (where μ and σ are the mean and standard deviation of X), and its approximate normality if n is large (ACMSM138)
- simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate the approximate standard normality of $\frac{\overline{X}-\mu}{s/\sqrt{n}}$ for large samples $(n \geq 30)$, where s is the sample standard deviation. (ACMSM139)

Confidence intervals for means:

- understand the concept of an interval estimate for a parameter associated with a random variable (ACMSM140)
- examine the approximate confidence interval $\left(\overline{\mathrm{X}} - \frac{\mathrm{zs}}{\sqrt{n}},\ \overline{\mathrm{X}} + \frac{\mathrm{zs}}{\sqrt{n}}\right)$, as an interval estimate for μ ,the population mean, where z is the appropriate quantile for the standard normal distribution (ACMSM141)
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ (ACMSM142)
- use $\overline{x}$ and s to estimate μ and σ, to obtain approximate intervals covering desired proportions of values of a normal random variable and compare with an approximate confidence interval for μ (ACMSM143)
- collect data and construct an approximate confidence interval to estimate a mean and to report on survey procedures and data quality. (ACMSM144)

Units 3 and 4 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge and understanding of concepts of functions, calculus, vectors and statistics in routine and non-routine problems in a variety of contexts • synthesises information to select and apply techniques in mathematics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical models to routine and non-routine problems in a variety of contexts • constructs mathematical proofs in a variety of contexts using a range of techniques • uses digital technologies effectively to graph, display and organise mathematical information to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of functions, calculus, vectors and statistics in routine and non-routine problems • synthesises information to select and apply techniques in mathematics to solve routine and non-routine problems • selects and applies mathematical models to routine and non-routine problems • constructs mathematical proofs in a variety of contexts and adapts previously seen mathematical proofs • uses digital technologies appropriately to graph, display and organise mathematical information to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of functions, calculus, vectors and statistics that apply to routine problems • selects and applies techniques in mathematics to solve routine problems • applies mathematical models to routine problems • constructs simple mathematical proofs and adapts previously seen mathematical proofs • uses digital technologies to graph, display and organise mathematical information to solve routine problems	• demonstrates knowledge of concepts of functions, calculus, vectors and statistics • uses simple techniques in mathematics in routine problems • demonstrates familiarity with mathematical models • reproduces previously seen mathematical proofs • uses digital technologies to display some mathematical information in routine problems	• demonstrates limited familiarity with simple concepts of functions, calculus, vectors and statistics • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical models • reproduces previously seen simple mathematical proofs • uses digital technologies for arithmetic calculations and to display limited mathematical information

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates succinct and reasoned mathematical and statistical judgments and arguments, including proofs, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates clear and reasoned mathematical and statistical judgments and arguments, including proofs, using appropriate language • interpret the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments, including simple proofs, using appropriate language • interprets the solutions to routine problems • describes the reasonableness of the results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical or symbolic form in routine problems • communicates mathematical and statistical arguments, including previously seen proofs, using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	• represents simple mathematical and statistical information in a structured context • communicates simple mathematical and statistical information using appropriate language • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

Specialist Mathematics Glossary

(Multiplicative) identity matrix

A (multiplicative)identity matrix is a square matrix in which all the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter *I*.

For example,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ are both identity matrices.

There is an identity matrix for each order of square matrix. When clarity is needed, the order is written with a subscript: I_n

Addition of matrices (See Matrix)

If A and B are matrices with the same dimensions and the entries of A are a_{ij} and the entries of B are b_{ij} then the entries of $A + B$ are $a_{ij} + b_{ij}$

For example if

$$\mathrm{A} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} \text{ and } \mathrm{B} = \begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 6 \end{bmatrix} \text{ then}$$

$$\mathrm{A} + \mathrm{B} = \begin{bmatrix} 7 & 2 \\ 2 & 4 \\ 2 & 10 \end{bmatrix}$$

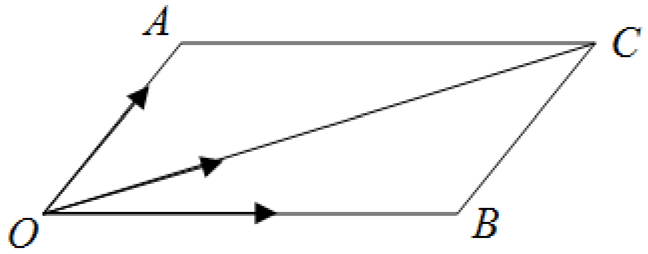
Addition of vectors

In component form if a = a_1i + a_2j + a_3k and b = b_1i + b_2j + b_3k then

a + b = (a_1 + b_1)i + (a_2 + b_2)j + (a_3+ b_3)k

Addition of vectors (see Vector for definition and notation)

Given vectors a and b let $\vec{OA}$ and $\vec{OB}$ be directed line segments that represent a and b. They have the same initial point *O*. The sum of $\vec{OA}$ and $\vec{OB}$ is the directed line segment $\vec{OC}$ where *C* is a point such that *OACB* is a parallelogram. This is known as the parallelogram rule.



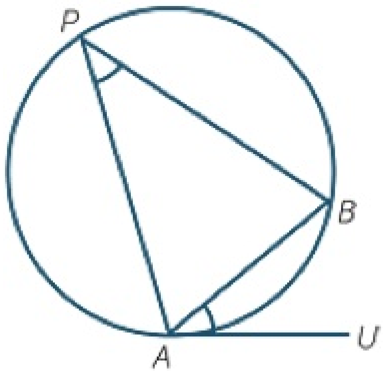
If a = (a_1, a_2) and b = (b_1, b_2) then a + b = $(a_1 + b_1, a_2 + b_2)$

In component form if a = a_1i + a_2j and b = b_1i + b_2j then

a + b = $(a_1 + b_1)$i + $(a_2 + b_2)$j

Alternate segment

The word 'alternate' means 'other'. The chord *AB* divides the circle into two segments and *AU* is tangent to the circle. Angle *APB* 'lies in' the segment on the other side of chord *AB* from angle *BAU.* We say that it is in the alternate segment.



Angle sum and difference identities

sin $(A + B)$ = sin A cos B + sin B cos A

sin$(A – B)$ = sin A cos B – sin B cos A

cos$(A + B)$ = cos A cos B – sin A sin B

cos$(A – B)$ = cos A cos B + sin A sin B

Argument (abbreviated arg)

If a complex number is represented by a point *P* in the complex plane then the argument of *z*, denoted arg *z*, is the angle θ that *OP* makes with the positive real axis O_x, with the angle measured anticlockwise from O_x. The principalvalue of the argument is the one in the interval (–π, π].

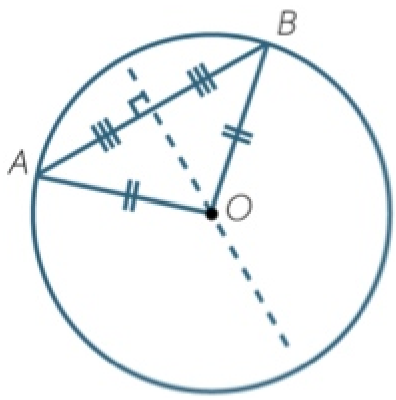
Arranging n objects in an ordered list

The number of ways to arrange n different objects in an ordered list is given by

$n(n-1)(n-2) \times \ldots \times 3 \times 2 \times 1 = n!$

Circle Theorems

Result 1



Let *AB* be a chord of a circle with centre *O*.

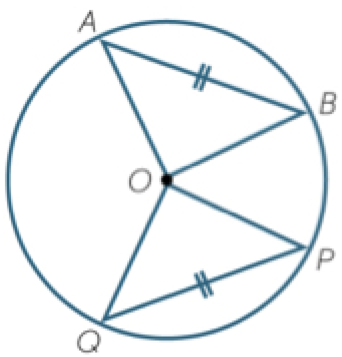
The following three lines coincide:

The bisector of the angle ∠*AOB* subtended at the centre by the chord.

The line segment (interval) joining *O* and the midpoint of the chord *AB*.

The perpendicular bisector of the chord *AB*.

Result 2

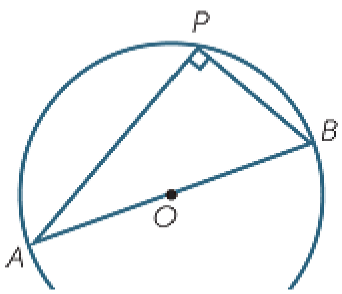


Equal chords of a circle subtend equal angles at the centre.

In the diagram shown ∠*AOB* = ∠*POQ*.

Result 3

An angle in a semicircle is a right angle.



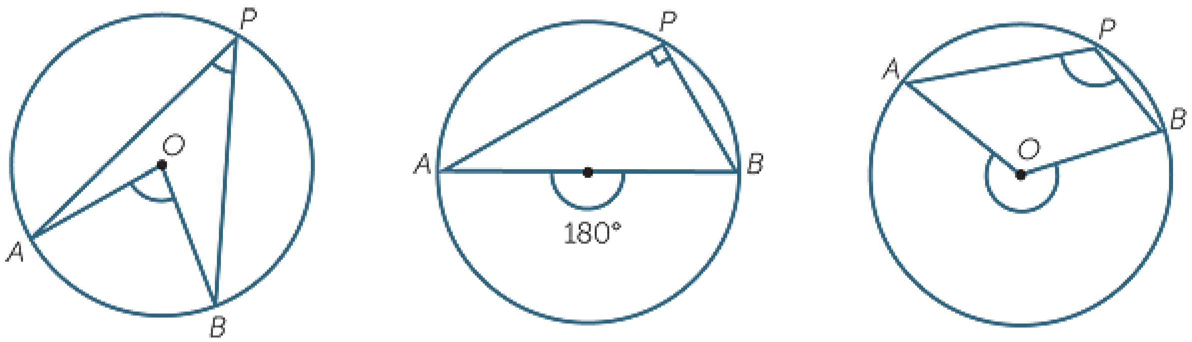
Let *AOB* be a diameter of a circle with centre *O*, and let *P* be any other point on the circle. The angle $\angle APB$ subtended at *P* by the diameter *AB* is called an angle in a semicircle.

Converse

The circle whose diameter is the hypotenuse of a right-angled triangle passes through all three vertices of the triangle.

Result 4

An angle at the circumference of a circle is half the angle subtended at the centre by the same arc. In the diagram shown $\angle AOB = 2\angle APB$

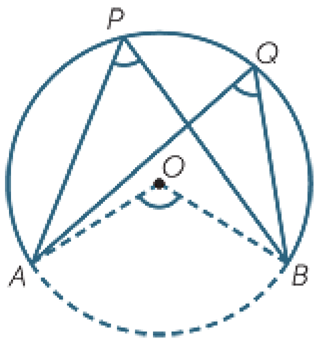


The arc *AB*subtends the angle $\angle AOB$ at the centre. The arc also subtends the angle $\angle APB$, called an angle at the circumference subtended by the arc *AB*.

Result 5

Two angles at the circumference subtended by the same arc are equal.

$\angle APB = \angle AQB$



In the diagram, the two angles $\angle APB$ and $\angle AQB$ are subtended by the same arc *AB*.

Result 6

The opposite angles of a cyclic quadrilateral are supplementary.

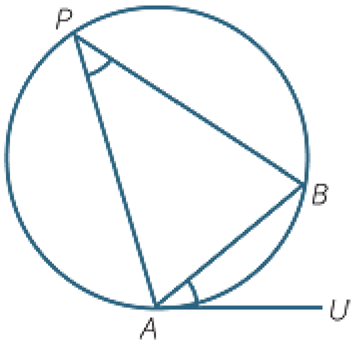
Converse

If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

Result 7

Alternate segment theorem

An angle between a chord and a tangent is equal to any angle in the alternate segment.



In the diagram $\angle BAU = \angle APB$.

Combinations (Selections)

The number of selections of n objects taken r at a time (that is, the number of ways of selecting r objects out of n) is denoted by $^nC_r = \binom{n}{r}$ and is equal to $\frac{n!}{r!(n-r)!}$

Complex arithmetic

If $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$

$z_1 + z_2 = x_1 + x_2 + (y_1 + y_2)\, i$

$z_1 - z_2 = x_1 - x_2 + (y_1 - y_2)\, i$

$z_1 \times z_2 = x_1 x_2 - y_1 y_2 + (x_1 y_2 + x_2 y_1)i$

$z_1 \times (0 + 0i) = 0$ Note: $0 + 0i$ is usually written as 0

$z_1 \times (1 + 0i) = z_1$ Note: $1 + 0i$ is usually written as 1

Complex conjugate

For any complex number $z = x + iy$, its conjugate is $\overline{z} = x - iy$. The following properties hold

$\overline{z_1 z_2} = \overline{z_1}\ \overline{z_2}$

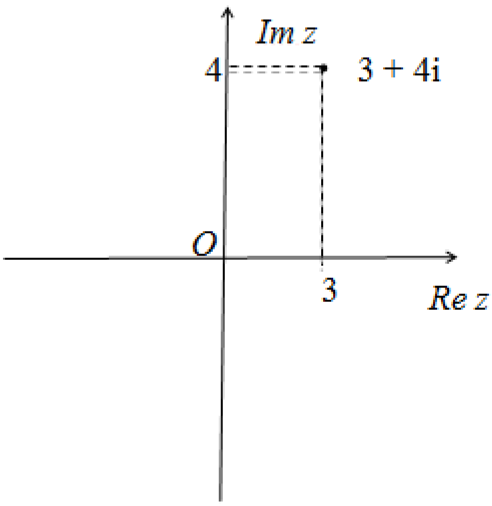
$\overline{z_1/z_2} = \overline{z_1}\ /\ \overline{z_2}$

$\overline{z} = [z]^2$

$z + \overline{z}$ is real

Complex plane (Argand plane)

The complex plane is a geometric representation of the complex numbers established by the real axis and the orthogonal imaginary axis. The complex plane is sometimes called the Argand plane.



Continuous random variable

A random variable X is called continuous if its set of possible values consists of intervals, and the chance that it takes any point value is zero (in symbols, if $P\,(X = x) = 0$ for every real number x). A random variable is continuous if and only if its cumulative probability distribution function can be expressed as an integral of a function.

Contradiction-Proof by

Assume the opposite (negation) of what you are trying to prove. Then proceed through a logical chain of argument till you reach a demonstrably false conclusion. Since all the reasoning is correct and a false conclusion has been reached the only thing that could be wrong is the initial assumption. Therefore the original statement is true.

For example: the result $\sqrt{2}$ is irrational can be proved in this way by first assuming $\sqrt{2}$ is rational.

The following are examples of results that are often proved by contradiction:

If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

If an interval (line segment) subtends equal angles at two points on the same side of the interval (line segment), then the two points and the endpoints of the interval are concyclic.

Implication: if P then Q Symbol: P ⇒ Q

Examples:

If a quadrilateral is a rectangle then the diagonals are of equal length and they bisect each other.

If $x = 2$ then $x^2 = 4$.

If an animal is a kangaroo then it is a marsupial.

If a quadrilateral is cyclic then the opposite angles are supplementary.

Converse of a statement

The converse of the statement 'If P then Q' is 'If Q then P' Symbolically the converse of P ⇒ Q is: Q ⇒ P or P ⇐ Q

The converse of a true statement need not be true.

Examples:

Statement: If a quadrilateral is a rectangle then the diagonals are of equal length and they bisect each other.

Converse statement: If the diagonals of a quadrilateral are of equal length and bisect each other then the quadrilateral is a rectangle. (In this case the converse is true.)

Statement: If $x = 2$ then $x^2 = 4$.

Converse statement: If $x^2 = 4$ then $x = 2$. (In this case the converse is false.)

Statement: If an animal is a kangaroo then it is a marsupial.

Converse statement: If an animal is a marsupial then it is a kangaroo. (In this case the converse is false.)

Statement: If a quadrilateral is cyclic then the opposite angles are supplementary.

Converse statement: If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic. (In this case the converse is true.)

Contrapositive

The contrapositive of the statement 'If P then Q' is 'If not Q then not P'. The contrapositive of a true statement is also true. (not Q is the negation of the statement Q)

Examples:

Statement: A rectangle is a quadrilateral that has diagonals of equal length and the diagonals bisect each other.

Contrapositive: If the diagonals of a quadrilateral are not of equal length or do not bisect each other then the quadrilateral is not a rectangle.

Statement: If $x = 2$ then $x^2 = 4$.

Contrapositive: If $x^2 \neq 4$ then $x \neq 2$.

Statement: A kangaroo is a marsupial.

Contrapositive: If an animal is not a marsupial then it is not a kangaroo.

Statement: The opposite angles of a cyclic quadrilateral are supplementary

Contrapositive: If the opposite angles of quadrilateral are not supplementary then the quadrilateral is not cyclic.

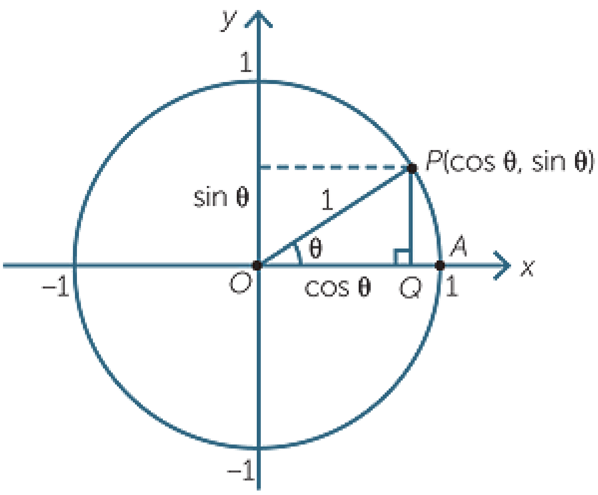
Cosine and sine functions

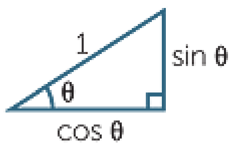
Since each angle θ measured anticlockwise from the positive *x*-axis determines a point P on the unit circle, we will define

the cosine of θ to be the x-coordinate of the point *P*

the sine of θ to be the y-coordinate of the point *P*

the tangent of θ is the gradient of the line segment *OP*





Counterexample

A Counterexample is an example that demonstrates that a statement is not true.

Examples:

Statement: If $x^2 = 4$ then $x = 2$.

Counterexample: $x = -2$ provides a counterexample.

Statement: If the diagonals of a quadrilateral intersect at right angles then the quadrilateral is a rhombus.

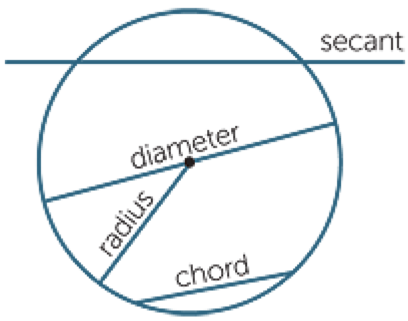
Counterexample: A kite with the diagonals not bisecting each other is not a rhombus. Such a kite provides a counterexample to the statement. The diagonals of a kite do intersect at right angles.

Statement: Every convex quadrilateral is a cyclic quadrilateral.

Counterexample: A parallelogram that is not a rectangle is convex, but not cyclic.

Cyclic quadrilateral

A cyclic quadrilateral is a quadrilateral whose vertices all lie on a circle.



Lines and line segments associated with circles

Any line segment joining a point on the circle to the centre is called a radius. By the definition of a circle, any two radii have the same length called the radius of the circle. Notice that the word 'radius' is used to refer both to these intervals and to the common length of these intervals.

An interval joining two points on the circle is called a chord.

A chord that passes through the centre is called a diameter. Since a diameter consists of two radii joined at their endpoints, every diameter has length equal to twice the radius. The word 'diameter' is use to refer both to these intervals and to their common length.

A line that cuts a circle at two distinct points is called a secant. Thus a chord is the interval that the circle cuts off a secant, and a diameter is the interval cut off by a secant passing through the centre of a circle.

De Moivre's Theorem

For all integers n, $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$.

Determinant of a 2 × 2 matrix (See Matrix)

If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the determinant of A denoted as det A = ad – bc.

If det $A \neq 0$,

the matrix A has an inverse.

the simultaneous linear equations $ax + by = e$ and $cx + dy = f$ have a unique solution.

The linear transformation of the plane, defined by A maps the unit square

$O(0, 0)$, $B(0,1)$, $C(1, 1)$, $D(1, 0)$ to a parallelogram $OB'C'D'$ of area |det A |.

The sign of the determinant determines the orientation of the image of a figure under the transformation defined by the matrix.

Dimension (or size) (See Matrix)

Two matrices are said to have the same dimensions (or size) if they have the same number of rows and columns.

For example, the matrices

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \, and \, \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

have the same dimensions. They are both 2 × 3 matrices.

An $m \times n$ matrix has m rows and n columns.

Double angle formula

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - tan^2 A}$$

Entries (Elements) of a matrix

The symbol a_{ij} represents the (i, j) entry which occurs in the i^{th} row and the j^{th} column.

For example a general 3 × 2 matrix is:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

and a32 is the entry in the third row and the second column.

Equivalent statements

Statements P and Q are equivalent if P ⇒ Q and Q ⇒ P. The symbol ⇔ is used. It is also written as P if and only if Q or P iff Q.

Examples:

A quadrilateral is a rectangle if and only if the diagonals of the quadrilateral are of equal length and bisect each other.

A quadrilateral is cyclic if and only if opposite angles are supplementary.

Negation

If P is a statement then the statement 'not P', denoted by ¬P is the negation of P. If P is the statement 'It is snowing.' then ¬P is the statement 'It is not snowing.'.

Imaginary part of a complex number

A complex number *z* may be written as *x* + *yi*, where *x* and *y* are real, and then *y* is the imaginary part of *z*. It is denoted by *Im* (*z*).

Implicit differentiation

When variables x and y satisfy a single equation, this may define y as a function of x even though there is no explicit formula for y in terms of x. Implicit differentiation consists of differentiating each term of the equation as it stands and making use of the chain rule. This can lead to a formula for $\frac{dy}{dx}$. For example,

if $x^2 + xy^3 - 2x + 3y = 0,$

then $2x + x\left(3y^2\right)\frac{dy}{dx} + y^3 - 2 + 3\frac{dy}{dx} = 0,$

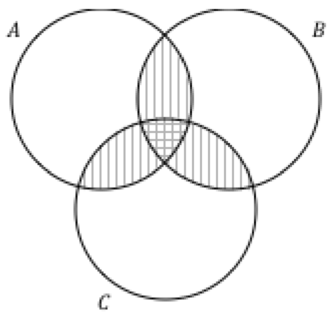
and so $\frac{dy}{dx} = \frac{2-2x-y^3}{3xy^2+3}$

Inclusion – exclusion principle

Suppose *A* and *B* are subsets of a finite set *X* then

|*A* ∪ *B* | = | *A* | + | *B* | – | *A* ∩ *B*|

Suppose *A*, *B* and *C* are subsets of a finite set *X* then



|*A* ∪ *B* ∪ *C*| = | *A* | + | *B* | + | *C* | – | *A* ∩ *B*| – | *A* ∩ *C*| – | *B* ∩ *C*| + | *A* ∩ *B* ∩ *C*|

This result can be generalised to 4 or more sets.

Independent and identically distributed observations

For independent observations, the value of any one observation has no effect on the chance of values for all the other observations. For identically distributed observations, the chances of the possible values of each observation are governed by the same probability distribution.

Integers

The integers are the numbers $\cdots, -3, \ -2, \ -1, \ 0, \ 1, \ 2, \ 3, \cdots$.

Modulus (Absolute value) of a complex number

If z is a complex number and $z = x + iy$ then the modulus of z is the distance of z from the origin in the Argand plane. The modulus of z denoted by $|z| = \sqrt{x^2 + y^2}$.

Leading diagonal

The leading diagonal of a square matrix is the diagonal which runs from the top left corner to the bottom right corner.

Linear momentum

The linear momentum p of a particle is the vector quantity p = mv where m is the mass and v is the velocity.

Linear transformation defined by a 2 × 2 matrix

The matrix multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax & + & by \\ cx & & dy \end{bmatrix}$$

defines a transformation $T(x, y) = (ax + by, cx + dy)$.

Linear transformations in 2 dimensions

A linear transformation in the plane is a mapping of the form

$T(x, y) = (ax + by, cx + dy)$.

A transformation T is linear if and only if

$T(\alpha(x_1, y_1) + \beta(x_2, y_2)) = \alpha T((x_1, y_1)) + \beta T(x_2, y_2))$.

Linear transformations include:

rotations around the origin

reflections in lines through the origin

dilations.

Translations are not linear transformations.

Logistic equation

The logistic equation has applications in a range of fields, including biology, biomathematics, economics, chemistry, mathematical psychology, probability, and statistics.

One form of this differential equation is:

$\frac{dy}{dt}$ = $ay - by^2$ (where $a > 0$ and $b > 0$)

The general solution of this is

$y = \frac{a}{b+C_e^{-at}}$ where C is an arbitrary constant.

Magnitude of a vector (see Vector for definition and notation)

The magnitude of a vector a is the length of any directed line segment that represents a. It is denoted by |a|.

Matrix (matrices)

A matrix is a rectangular array of elements or entries displayed in rows and columns.

For example,

$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ are both matrices.

Matrix A is said to be a 3 × 2 matrix (three rows and two columns) while B is said to be a 2 × 3 matrix (two rows and three columns).

A square matrix has the same number of rows and columns.

A column matrix (or vector) has only one column.

A row matrix (or vector) has only one row.

Matrix algebra of 2 × 2 matrices

If A, B and C are 2 × 2 matrices, I the 2 × 2 (multiplicative) identity matrix and O the 2 × 2 zero matrix then:

$A + B = B + A$ (commutative law for addition)

$(A + B) + C = A + (B + C)$ (associative law for addition)

$A + O = A$ (additive identity)

$A + (–A) = O$ (additive inverse)

$(AB)C = A(BC)$ (associative law for multiplication)

$AI = A = IA$ (multiplicative identity)

$A(B + C) = AB + AC$ (left distributive law)

$(B + C)A = BA + CA$ (right distributive law)

Matrix multiplication

Matrix multiplication is the process of multiplying a matrix by another matrix. The product *AB* of two matrices *A* and *B* with dimensions$m \times n$ and $p \times q$ is defined if $n = p$. If it is defined, the product *AB* is an $m \times q$ matrix and it is computed as shown in the following example.

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} 6 & 10 \\ 11 & 3 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 94 & 34 \\ 151 & 63 \end{bmatrix}$$

The entries are computed as shown $1 \times 6 + 8 \times 11 + 0 \times 12 = 94$

$1 \times 10 + 8 \times 3 + 0 \times 4 = 34$

$2 \times 6 + 5 \times 11 + 7 \times 12 = 151$

$2 \times 10 + 5 \times 3 + 7 \times 4 = 63$

The entry in row *i* and column *j* of the product *AB* is computed by 'multiplying' row *i* of *A* by column *j* of B as shown.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & ba_{23} \end{bmatrix}$ then

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{12}b_{12} + a_{22}b_{22} & a_{12}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$$

Modulus of a complex number

(Modulus of a complex number Unit 2)

Multiplication by a scalar

Let a be a non-zero vector and *k* a positive real number (scalar) then the scalar multiple of a by *k* is the vector *ka* which has magnitude |*k* ||a| and the same direction as a. If *k* is a negative real number, then *k* a has magnitude | *k* | |a| and but is directed in the opposite direction to a. (see negative of a vector)

Some properties of scalar multiplication are:

k (a + b) = *k* a + *k* b

h(*k* (a)) = (*hk*)a

1a = a

Multiplication principle

Suppose a choice is to be made in two stages. If there are a choices for the first stage and b choices for the second stage, no matter what choice has been made at the first stage, then there are *ab* choices altogether. If the choice is to be made in *n* stages and if for each *i,* there are a_i choices for the i^{th} stage then there are $a_1a_2 \ldots a_n$ choices altogether.

Multiplicative inverse of a square matrix

The inverse of a square matrix A is written as A^{-1} and has the property that

$AA^{-1} = A^{-1}A = I$

Not all square matrices have an inverse. A matrix that has an inverse is said to be invertible.

multiplicative inverse of a 2 × 2 matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\boldsymbol{is}\ \boldsymbol{A}^{-1}\ \frac{1}{det\boldsymbol{A}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, when det A ≠ 0.

Scalar multiplication (matrices)

Scalar multiplication is the process of multiplying a matrix by a scalar (number).

For example, forming the product:

$$10 \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 0 & 30 \\ 10 & 40 \end{bmatrix}$$

is an example of the process of scalar multiplication.

In general for the matrix A with entries a_{ij} the entries of kA are ka_{ij}.

Negative of a vector (see Vector for definition and notation)

Given a vector a, let $\vec{AB}$ be a directed line segment representing a. The negative of a, denoted by –a, is the vector represented by $\vec{BA}$. The following are properties of vectors involving negatives:

a + (–a) = (–a) + a = 0

–(–a) = a

Pascal's triangle

Pascal's triangle is an arrangement of numbers. In general the n^{th} row consists of the binomial Coefficients

${}^{n}C_{r}$ or $\binom{n}{r}$ with the r = 0, 1,…, n

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

In Pascal's triangle any term is the sum of the two terms 'above' it.

For example 10 = 4 + 6.

Identities include:

The recurrence relation, ${}^{n}C_{k} = {}^{n-1}C_{k-1} + {}^{n-1}C_{k}$

${}^{n}C_{k} = \frac{n}{k}\,{}^{n-1}C_{k-1}$

Permutations

A permutation of n objects is an arrangement or rearrangement of n objects (order is important). The number of arrangements of n objects is $n!$ The number of permutations of n objects taken r at a time is denoted ${}^{n}P_{r}$ and is equal to

$n(n-1)\ldots(n-r+1) = \frac{n!}{(n-r)!}$

Pigeon-hole principle

If there are n pigeon holes and $n + 1$ pigeons to go into them, then at least one pigeon hole must get 2 or more pigeons.

Polar form of a complex number

For a complex number z, let θ = arg z. Then $z = r(\cos\theta + i\sin\theta)$ is the polar form of z.

Precision

Precision is a measure of how close an estimator is expected to be to the true value of the parameter it purports to estimate.

Prime numbers

A prime number is a positive integer greater than 1 that has no positive integer factor other 1 and itself. The first few prime numbers are $2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\cdots.$

Principle of mathematical induction

Let there be associated with each positive integer n, a proposition $P(n)$.

If

$P(1)$ is true, and

for all k, $P(k)$ is true implies $P(k + 1)$ is true,

then $P(n)$ is true for all positive integers n.

Probability density function

The probability density function (pdf) of a continuous random variable is the function that when integrated over an interval gives the probability that the continuous random variable having that pdf, lies in that interval. The probability density function is therefore the derivative of the (cumulative probability) distribution function.

Products as sums and differences

$$\cos A \cos B = \frac{1}{2}\left(\cos\left(A - B\right) + \cos\left(A + B\right)\right)$$

$$\sin A \sin B = \frac{1}{2}\left(\cos\left(A - B\right) - \cos\left(A + B\right)\right)$$

$$\sin A \cos B = \frac{1}{2}\left(\sin\left(A + B\right) + \sin\left(A - B\right)\right)$$

$$\cos A \sin B = \frac{1}{2}\left(\sin\left(A + B\right) - \sin\left(A - B\right)\right)$$

Properties of vector addition:

a + b = b + a (commutative law)

(a + b) + c = a + (b + c) (associative law)

a + 0 = 0 + a = a

a + (–a) = 0

Pythagorean identities

$\cos^2 A + \sin^2 A = 1$

$\tan^2 A + 1 = \sec^2 A$

$\cot^2 A + 1 = \mathrm{cosec}^2\, A$

Quantifiers

For all (For each)

Symbol ∀

For all real numbers x, $x^2 \geq 0$. (∀ real numbers x, $x^2 \geq 0$.)

For all triangles the sum of the interior angles is 180^o .(∀ triangles the sum of the interior angles is 180^o.)

For each diameter of a given circle each angle subtended at the circumference by that diameter is a right angle.

There exists

Symbol ∃

There exists a real number that is not positive (∃ a real number that is not positive.)

There exists a prime number that is not odd. (∃ a prime number that is not odd.)

There exists a natural number that is less than 6 and greater than 3.

There exists an isosceles triangle that is not equilateral.

The quantifiers can be used together.

For example: $\forall\ x \geq 0,\ \exists\ y \geq 0$ such that $y^2 = x$.

Random sample

A random sample is a set of data in which the value of each observation is governed by some chance mechanism that depends on the situation. The most common situation in which the term “random sample” is used refers to a set of independent and identically distributed observations.

Sample mean

The arithmetic average of the sample values

Rational function

A rational function is a function such that $f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials. Usually $g(x)$ and $h(x)$ are chosen so as to have no common factor of degree greater than or equal to 1, and the domain of f is usually taken to be

$R\backslash\{x: h(x) = 0\}$.

Rational numbers

A real number is rational if it can be expressed as a quotient of two integers. Otherwise it is called irrational.

Irrational numbers can be approximated as closely as desired by rational numbers, and most electronic calculators use a rational approximation when performing calculations involving an irrational number.

Real numbers

The numbers generally used in mathematics, in scientific work and in everyday life are the real numbers. They can be pictured as points on a number line, with the integers evenly spaced along the line, and a real number a to the right of a real number b if $a > b$.

A real number is either rational or irrational. The set of real numbers consists of the set of all rational and irrational numbers.

Every real number has a decimal expansion. Rational numbers are the ones whose decimal expansions are either terminating or eventually recurring.

Real part of a complex number

A complex number *z* may be written as $x + yi$, where *x* and *y* are real, and then *x* is the real part of *z*. It is denoted by $Re\,(z)$.

Reciprocal trigonometric functions

$\sec A = \frac{1}{\cos A}$, $\cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}$, $\sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}$, $\sin A \neq 0$

Root of unity (nth root of unity)

A complex number *z* such that $z^n = 1$

The n^{th} roots of unity are:

$\cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}$ where $k = 0, 1, 2, \ldots, n - 1$.

The points in the complex plane representing roots of unity lie on the unit circle.

The cube roots of unity are

$$z_1 = 1,\ z_2 = \frac{1}{2}\left(-1 + i\sqrt{3}\right),\quad z_3 = \frac{1}{2}\left(-1 - i\sqrt{3}\right)$$

Note $z_3 = \overline{z_2}$ and $z_3 = \frac{1}{z_2}$ and $z_2 z_3 = 1$

Scalar product

If a = (a_1, a_2, a_3) and b = (b_1, b_2, b_3) then the scalar product a.b is the real number

$a_1b_1 + a_2b_2 + a_3b_3$.

When expressed in i, j, k notation, a = a_1i + a_2j + a_3k and b = b_1i + b_2j + b_3k then

a.b = $a_1b_1 + a_2b_2 + a_3b_3$

Scalar product (see Vector for definition and notation)

a = (a_1, a_2) and b = (b_1, b_2) then the scalar product a.b is the real number

a_1 b_1 + a_2 b_2. The geometrical interpretation of this number is a.b =|a||b|cos (θ) where θ is the angle 'between' a and b

When expressed in i, j, notation, if a = a_1i + a_2j and b = b_1i + b_2j then

a.b = a_1 b_1 + a_2 b_2

Note |a| = $\sqrt{a.a}$

Separation of variables

Differential equations of the form $\frac{dy}{dx} = g(x)h\left(y\right)$ can be rearranged as long as $h(y) \neq 0$ to obtain

$$\frac{1}{h\left(y\right)} \frac{dy}{dx} = g\left(x\right)$$

Singular matrix

A matrix is singular if det A = 0. A singular matrix does not have a multiplicative inverse.

Slope field

Slope field (direction or gradient field) is a graphical representation of the solutions of a linear first-order differential equation in which the derivative at a given point is represented by a line segment of the corresponding slope

Subtraction of vectors (see Vector for definition and notation)

a – b = a + (–b)

Unit vector (see Vector for definition and notation)

A unit vector is a vector with magnitude 1. Given a vector a the unit vector in the same direction as a is $\frac{1}{|a|}$

a. This vector is often denoted as $\hat{a}$.

Vector equation of a plane

Let a be a position vector of a point A in the plane, and n a normal vector to the plane . Then the plane consists of all points P whose position vector p satisfies

(p – a).n = 0. This equation may also be written as p.n =a.n, a constant.

(If the the normal vector n is the vector (l, m, n) in ordered triple notation and the scalar product

a.n = k, this gives the Cartesian equation $lx + my + nz = k$ for the plane)

Vector equation of a straight line

Let a be the position vector of a point on a line *l*, and u any vector with direction along the line. The line consists of all points *P* whose position vector p is given by

p = a + *t*u for some real number *t*.

(Given the position vectors of two points on the plane a and b the equation can be written as

p = a + *t*(b – a) for some real number *t*.)

Vector function

In this course a vector function is one that depends on a single real number parameter t, often representing time, producing a vector *r*(*t*) as the result. In terms of the standard unit vectors i, j, k of three dimensional space, the vector-valued functions of this specific type are given by expressions such as

r(*t*) = *f*(*t*)i + *g*(*t*)j + *h*(*t*)k

where *f*, *g* and *h* are real valued functions giving coordinates.

Vector product (Cross product)

When expressed in i, j, k notation, a = a_1i + a_2j + a_3k and b = b_1i + b_2j + b_3k then

a × b = $(a_2b_3 - a_3b_2)$i + $(a_3b_1 - a_1b_3)$j + $(a_1b_2 - a_2b_1)$k

The cross product has the following geometric interpretation. Let a and b be two non- parallel vectors then |a × b| is the area of the parallelogram defined by a and b and

a × b is normal to this parallelogram.

(The cross product of two parallel vectors is the zero vector.)

The inverse sine function, $y = \sin^{-1}x$

If the domain for the sine function is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ a one to one function is formed and so an inverse function exists.

The inverse of this restricted sine function is denoted by $\sin^{-1}$ and is defined by:

$\sin^{-1}: [-1, 1] \to R$, $\sin^{-1}x = y$ where $\sin y = x$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}$ is also denoted by arcsin.

The inverse cosine function, $y = \cos^{-1}x$

If the domain of the cosine function is restricted to $[0, \pi]$ a one to one function is formed and so the inverse function exists.

$\cos^{-1}x$, the inverse function of this restricted cosine function, is defined as follows:

$\cos^{-1}: [-1, 1] \to R$, $\cos^{-1}x = y$ where $\cos y = x$, $y \in [0, \pi]$

$\cos^{-1}$ is also denoted by arccos.

The inverse tangent function, $y = \tan^{-1}x$

If the domain of the tangent function is restricted to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ a one to one function is formed and so the inverse function exists.

$\mathrm{Tan}^{-1}: R \to R$, $\tan^{-1}x = y$ where $\tan y = x$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Tan^{-1} is also denoted by arctan.

Vector projection (see Vector for definition and notation)

Let a and b be two vectors and write θ for the angle between them. The projection of a vector a on a vector b is the vector

$|a| \cos\theta\, \hat{b}$ where $\hat{b}$ is the unit vector in the direction of b.

The projection of a vector a on a vector b is (a. $\hat{b}$) $\hat{b}$ where $\hat{b}$ is the unit vector in the direction of b. This projection is also given by the formula $\frac{a.b}{a.b}\, b$.

In Physics the name vector is used to describe a physical quantity like velocity or force that has a magnitude and direction.

A vector is an entity a which has a given length (magnitude) and a given direction. If $\vec{AB}$ is a directed line segment with this length and direction, then we say that $\vec{AB}$represents a.

If $\vec{AB}$ and $\vec{CD}$ represent the same vector, they are parallel and have the same length.

The zero vector is the vector with length zero.

In two dimensions, every vector can be represented by a directed line segment which begins at the origin. For example, the vector $\vec{BC}$ from B(1,2) to C(5,7) can be represented by the directed line segment $\vec{OA}$, where A is the point (4,5). The ordered pair notation for a vector uses the co-ordinates of the end point of this directed line segment beginning at the origin to denote the vector, so

$\vec{BC}$ = (4,5) in ordered pair notation. The same vector can be represented in column vector notation as .

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Vectors in three-dimensions

(See Vectors in Unit 2)

Whole numbers

A whole number is a non–negative integer, that is, one of the numbers $0,\ 1,\ 2,\ 3, \cdots$,

Zero matrix

A zero matrix is a matrix if all of its entries are zero. For example:

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ And $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ are zero matrices.

There is a zero matrix for each size of matrix. When clarity is needed we write for the *n* × *m* zero matrix. $\mathbf{O}_{n \times m}$

Achievement Standards Glossary

Glossary

Abstract

Abstract scenario: a scenario for which there is no concrete referent provided.

Account

Account for: provide reasons for (something).

Give an account of: report or describe an event or experience.

Taking into account: considering other information or aspects.

Analyse

Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences.

Apply

Use, utilise or employ in a particular situation.

Assess

Determine the value, significance or extent of (something).

Coherent

Orderly, logical, and internally consistent relation of parts.

Communicates

Conveys knowledge and/or understandings to others.

Compare

Estimate, measure or note how things are similar or dissimilar.

Complex

Consisting of multiple interconnected parts or factors.

Considered

Formed after careful thought.

Critically analyse

Examine the component parts of an issue or information, for example the premise of an argument and its plausibility, illogical reasoning or faulty conclusions

Critically evaluate

Evaluation of an issue or information that includes considering important factors and available evidence in making critical judgement that can be justified.

Deduce

Arrive at a conclusion by reasoning.

Demonstrate
Give a practical exhibition as an explanation.

Describe
Give an account of characteristics or features.

Design
Plan and evaluate the construction of a product or process.

Develop
In history: to construct, elaborate or expand.

In English: begin to build an opinion or idea.

Discuss
Talk or write about a topic, taking into account different issues and ideas.

Distinguish
Recognise point/s of difference.

Evaluate
Provide a detailed examination and substantiated judgement concerning the merit, significance or value of something.

In mathematics: calculate the value of a function at a particular value of its independent variables.

Explain
Provide additional information that demonstrates understanding of reasoning and/or application.

Familiar
Previously encountered in prior learning activities.

Identify
Establish or indicate who or what someone or something is.

Integrate
Combine elements.

Investigate
Plan, collect and interpret data/information and draw conclusions about.

Justify
Show how an argument or conclusion is right or reasonable.

Locate
Identify where something is found.

Manipulate
Adapt or change.

Non-routine

Non-routine problems: Problems solved using procedures not previously encountered in prior learning activities.

Reasonableness

Reasonableness of conclusions or judgements: the extent to which a conclusion or judgement is sound and makes sense

Reasoned

Reasoned argument/conclusion: one that is sound, well-grounded, considered and thought out.

Recognise

Be aware of or acknowledge.

Relate

Tell or report about happenings, events or circumstances.

Represent

Use words, images, symbols or signs to convey meaning.

Reproduce

Copy or make close imitation.

Responding

In English: When students listen to, read or view texts they interact with those texts to make meaning. Responding involves students identifying, selecting, describing, comprehending, imagining, interpreting, analysing and evaluating.

Routine problems

Routine problems: Problems solved using procedures encountered in prior learning activities.

Select

Choose in preference to another or others.

Sequence

Arrange in order.

Solve

Work out a correct solution to a problem.

Structured

Arranged in a given organised sequence.

In Mathematics: When students provide a structured solution, the solution follows an organised sequence provided by a third party.

Substantiate

Establish proof using evidence.

Succinct

Written briefly and clearly expressed.

Sustained

Consistency maintained throughout.

Synthesise

Combine elements (information/ideas/components) into a coherent whole.

Understand

Perceive what is meant, grasp an idea, and to be thoroughly familiar with.

Unfamiliar

Not previously encountered in prior learning activities.