The Australian Curriculum

Subjects	Mathematical Methods
Units	Unit 1, Unit 2, Unit 3 and Unit 4
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Mathematical Methods

Mathematical Methods

Rationale and Aims

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real-world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The major themes of Mathematical Methods are calculus and statistics. They include as necessary prerequisites studies of algebra, functions and their graphs, and probability. They are developed systematically, with increasing levels of sophistication and complexity. Calculus is essential for developing an understanding of the physical world because many of the laws of science are relationships involving rates of change. Statistics is used to describe and analyse phenomena involving uncertainty and variation. For these reasons this subject provides a foundation for further studies in disciplines in which mathematics and statistics have important roles. It is also advantageous for further studies in the health and social sciences. In summary, the subject Mathematical Methods is designed for students whose future pathways may involve mathematics and statistics and their applications in a range of disciplines at the tertiary level.

For all content areas of Mathematical Methods, the proficiency strands of the F-10 curriculum are still applicable and should be inherent in students' learning of this subject. These strands are Understanding, Fluency, Problem solving and Reasoning, and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as calculating derivatives and integrals, or solving quadratic equations, and frees up working memory for more complex aspects of problem solving. The ability to transfer skills to solve problems based on a wide range of applications is a vital part of mathematics in this subject. Because both calculus and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout this subject.

Mathematical Methods is structured over four units. The topics in Unit 1 build on students' mathematical experience. The topics 'Functions and graphs', 'Trigonometric functions' and 'Counting and probability' all follow on from topics in the F-10 curriculum from the strands, Number and Algebra, Measurement and Geometry and Statistics and Probability. In Mathematical Methods there is a progression of content and applications in all areas. For example, in Unit 2 differential calculus is introduced, and then further developed in Unit 3 where integral calculus is introduced. Discrete probability distributions are introduced in Unit 3, and then continuous probability distributions and an introduction to statistical inference conclude Unit 4.

Aims

Mathematical Methods aims to develop students':

- understanding of concepts and techniques drawn from algebra, the study of functions, calculus, probability and statistics
- ability to solve applied problems using concepts and techniques drawn from algebra, functions, calculus, probability and statistics
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Organisation

Overview of senior secondary Australian Curriculum

ACARA has developed senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications. The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

The senior secondary Australian Curriculum specifies content and achievement standards for each senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) expected of students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a 'school year' of senior secondary studies (approximately 50–60 hours duration including assessment and examinations). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, year-long), or as four units over two years.

State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into their courses. They will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and Board. They will determine the assessment and certification specifications for their local courses that integrate the Australian Curriculum content and achievement standards and any additional information, guidelines and rules to satisfy local requirements including advice on entry and exit points and credit for completed study.

The senior secondary Australian Curriculum for each subject should not, therefore, be read as a course of study. Rather, it is presented as content and achievement standards for integration into state and territory courses.

Senior secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics, with each subject organized into four units. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically, to make informed decisions. It provide students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. This subject provides the opportunity for students to prepare for post-school options of employment and further training.

General Mathematics focuses on using the techniques of discrete mathematics to solve problems in contexts that include financial modelling, network analysis, route and project planning, decision making, and discrete growth and decay. It provides an opportunity to analyse and solve a wide range of geometrical problems in areas such as measurement, scaling, triangulation and navigation. It also provides opportunities to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve comparing groups, investigating associations and analysing time series.

Mathematical Methods focuses on the development of the use of calculus and statistical analysis. The study of calculus in Mathematical Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Mathematical Methods develops the ability to describe and analyse phenomena involving uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. Specialist Mathematics is the only mathematics subject that cannot be taken as a stand-alone subject.

Structure of Mathematical Methods

Mathematical Methods is organised into four units. The topics broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. The probability and statistics topics lead to an introduction to statistical inference.

Unit 1	Unit 2	Unit 3	Unit 4
Functions and graphs Trigonometric functions Counting and probability	Exponential functions Arithmetic and geometric sequences and series Introduction to differential calculus	Further differentiation and applications Integrals Discrete random variables	The logarithmic function Continuous random variables and the normal distribution Interval estimates for proportions

Units

Unit 1 begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of functions and calculus. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of probability and statistics begins in this unit with a review of the fundamentals of probability, and the introduction of the concepts of conditional probability and independence. The study of the trigonometric functions begins with a consideration of the unit circle using degrees and the trigonometry of triangles and its application. Radian measure is introduced, and the graphs of the trigonometric functions are examined and their applications in a wide range of settings are explored.

In Unit 2, exponential functions are introduced and their properties and graphs examined. Arithmetic and geometric sequences and their applications are introduced and their recursive definitions applied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically (by calculating difference quotients), geometrically (as slopes of chords and tangents), and algebraically. This first calculus topic concludes with derivatives of polynomial functions, using simple applications of the derivative to sketch curves, calculate slopes and equations of tangents, determine instantaneous velocities, and solve optimisation problems.

In Unit 3, the study of calculus continues by introducing the derivatives of exponential and trigonometric functions and their applications, as well as some basic differentiation techniques and the concept of a second derivative, its meaning and applications. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The unit includes integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. Discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference.

In Unit 4, the logarithmic function and its derivative are studied. Continuous random variables are introduced and their applications examined. Probabilities associated with continuous distributions are calculated using definite integrals. In this unit students are introduced to one of the most important parts of statistics, namely statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit, inference is restricted to estimating proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: 'Concepts and Techniques' and 'Reasoning and Communication'. These two dimensions reflect students' understanding and skills in the study of mathematics.

Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained *a priori* without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

- Given a specified context (as described in the curriculum content)
- With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
- Students perform a specified action (described through a verb)
- In relation to what is valued in the curriculum (specified as the object or subject)
- With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as 'analyse' and 'describe' have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to be able to choose when or when not to use some form of technology and to be able to work flexibly with technology are important skills in these subjects.

Links to Foundation to Year 10

In Mathematical Methods, there is a strong emphasis on mutually reinforcing proficiencies in Understanding, Fluency, Problem solving and Reasoning. Students gain fluency in a variety of mathematical and statistical skills, including algebraic manipulations, constructing and interpreting graphs, calculating derivatives and integrals, applying probabilistic models, estimating probabilities and parameters from data, and using appropriate technologies. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving. In order to study Mathematical Methods, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended for the study of Mathematical Methods:

- ACMNA264: Define rational and irrational numbers, and perform operations with surds and fractional indices
- ACMNA269: Factorise monic and non-monic quadratic expressions, and solve a wide range of quadratic equations derived from a variety of contexts
- ACMSP278: Calculate and interpret the mean and standard deviation of data, and use these to compare datasets.

Representation of General capabilities

The seven general capabilities of *Literacy*, *Numeracy*, *Information and Communication technology (ICT) capability*, *Critical and creative thinking*, *Personal and social capability, Ethical understanding,* and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains financial applications of Mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age. Students will also learn about the probability of certain events occurring and will therefore be well equipped to make informed decisions.

ICT in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and creative thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and social capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical undertanding in Mathematics

In the senior years students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical behaviour. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person’s environment and language.

Representation of Cross-curriculum priorities

The senior secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students’ understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples histories and cultures.

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. Through analysis of relevant data, students are provided with opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Each of the senior Mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving.Therefore, teachers are encouraged to select contexts for discussion connected with sustainability. Through analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Unit 1

Unit Description

This unit begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of calculus. The basic trigonometric functions are then introduced. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of inferential statistics begins in this unit with a review of the fundamentals of probability and the introduction of the concepts of conditional probability and independence. Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in algebra, functions, graphs, trigonometric functions and probability
- solve problems using algebra, functions, graphs, trigonometric functions and probability
- apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions and probability
- interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
- communicate their arguments and strategies when solving problems.

Content Descriptions

Topic 1: Functions and graphs

Lines and linear relationships:

- determine the coordinates of the midpoint of two points (ACMMM001)
- examine examples of direct proportion and linearly related variables (ACMMM002)
- recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient (ACMMM003)
- find the equation of a straight line given sufficient information; parallel and perpendicular lines (ACMMM004)
- solve linear equations. (ACMMM005)

Review of quadratic relationships:

- examine examples of quadratically related variables (ACMMM006)
- recognise features of the graphs of $y = x^2$, $y = a(x-b)^2 + c$, and $y = a(x-b)(x-c)$, including their parabolic nature, turning points, axes of symmetry and intercepts (ACMMM007)
- solve quadratic equations using the quadratic formula and by completing the square (ACMMM008)
- find the equation of a quadratic given sufficient information (ACMMM009)
- find turning points and zeros of quadratics and understand the role of the discriminant (ACMMM010)
- recognise features of the graph of the general quadratic $y = ax^2 + bx + c$. (ACMMM011)

Inverse proportion:

- examine examples of inverse proportion (ACMMM012)
- recognise features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$, including their hyperbolic shapes, and their asymptotes. (ACMMM013)

Powers and polynomials:

- recognise features of the graphs of $y = x^n$ for $n \in \boldsymbol{N},\ \ n = -1 \text{ and } n = ½$, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM014)
- identify the coefficients and the degree of a polynomial (ACMMM015)
- expand quadratic and cubic polynomials from factors (ACMMM016)
- recognise features of the graphs of $y = x^3$, $y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$ (ACMMM017)
- factorise cubic polynomials in cases where a linear factor is easily obtained (ACMMM018)
- solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained. (ACMMM019)

Graphs of relations:

- recognise features of the graphs of $x^2 + y^2 = r^2$ and $(x-a)^2 + (y-b)^2 = r^2$, including their circular shapes, their centres and their radii (ACMMM020)
- recognise features of the graph of $y^2 = x$ including its parabolic shape and its axis of symmetry. (ACMMM021)

Functions:

- understand the concept of a function as a mapping between sets, and as a rule or a formula that defines one variable quantity in terms of another (ACMMM022)
- use function notation, domain and range, independent and dependent variables (ACMMM023)
- understand the concept of the graph of a function (ACMMM024)
- examine translations and the graphs of $y = f(x) + a$ and $y = f(x + b)$ (ACMMM025)
- examine dilations and the graphs of $y = cf(x)$ and $y = f(kx)$ (ACMMM026)
- recognise the distinction between functions and relations, and the vertical line test. (ACMMM027)

Topic 2: Trigonometric functions

Cosine and sine rules:

- review sine, cosine and tangent as ratios of side lengths in right-angled triangles (ACMMM028)
- understand the unit circle definition of $\cos\theta,\ \sin\theta$ and $\tan\theta$ and periodicity using degrees (ACMMM029)
- examine the relationship between the angle of inclination of a line and the gradient of that line (ACMMM030)
- establish and use the sine and cosine rules and the formula $Area = \frac{1}{2}bc\sin A$for the area of a triangle. (ACMMM031)

Circular measure and radian measure:

- define and use radian measure and understand its relationship with degree measure (ACMMM032)
- calculate lengths of arcs and areas of sectors in circles. (ACMMM033)

Trigonometric functions:

- understand the unit circle definition of $\cos\theta,\ \sin\theta$ and $\tan\theta$ and periodicity using radians (ACMMM034)
- recognise the exact values of $\sin\theta,\ \cos\theta \text{ and } \tan\theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (ACMMM035)
- recognise the graphs of $y = \sin x,\ y = \cos x,\ \text{and } y = \tan x$ on extended domains (ACMMM036)
- examine amplitude changes and the graphs of $y = a\sin x$ and $y = a\cos x$ (ACMMM037)
- examine period changes and the graphs of $y = \sin bx,\ \ y = \cos bx$, and $y = \tan\ bx$ (ACMMM038)
- examine phase changes and the graphs of $y = \sin(x + c),\ \ y = \cos(x + c)$ and (ACMMM039)
- $y = \tan\ (x + c)$ and the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin x$ (ACMMM040)
- prove and apply the angle sum and difference identities (ACMMM041)
- identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems (ACMMM042)
- solve equations involving trigonometric functions using technology, and algebraically in simple cases. (ACMMM043)

Topic 3: Counting and probability

Combinations:

- understand the notion of a combination as an unordered set of r objects taken from a set of n distinct objects (ACMMM044)
- use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects (ACMMM045)

- expand $(x+y)^n$ for small positive integers n (ACMMM046)
- recognise the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $(x+y)^n$) (ACMMM047)
- use Pascal's triangle and its properties. (ACMMM048)

Language of events and sets:

- review the concepts and language of outcomes, sample spaces and events as sets of outcomes (ACMMM049)
- use set language and notation for events, including $\overline{A}$ (or $A^{'}$) for the complement of an event A, $A?B$ for the intersection of events A and B, and $A?B$ for the union, and recognise mutually exclusive events (ACMMM050)
- use everyday occurrences to illustrate set descriptions and representations of events, and set operations. (ACMMM051)

Review of the fundamentals of probability:

- review probability as a measure of 'the likelihood of occurrence' of an event (ACMMM052)
- review the probability scale: $0 \le P(A) \le 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty (ACMMM053)
- review the rules: $P\left(\overline{A}\right) = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (ACMMM054)
- use relative frequencies obtained from data as point estimates of probabilities. (ACMMM055)

Conditional probability and independence:

- understand the notion of a conditional probability and recognise and use language that indicates conditionality (ACMMM056)
- use the notation $P(A|B)$ and the formula $P(A \cap B) = P(A|B)P(B)$ (ACMMM057)
- understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$ (ACMMM058)
- establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence (ACMMM059)
- use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events. (ACMMM060)

Unit 2

Unit Description

The algebra section of this unit focuses on exponentials and logarithms. Their graphs are examined and their applications in a wide range of settings are explored. Arithmetic and geometric sequences are introduced and their applications are studied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an ‘instantaneous rate of change’. These concepts are reinforced numerically, by calculating difference quotients both geometrically, as slopes of chords and tangents, and algebraically. Calculus is developed to study the derivatives of polynomial functions, with simple applications of the derivative to curve sketching, calculating slopes and equations of tangents, determining instantaneous velocities and solving optimisation problems.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques used in algebra, sequences and series, functions, graphs and calculus
- solve problems in algebra, sequences and series, functions, graphs and calculus
- apply reasoning skills in algebra, sequences and series, functions, graphs and calculus
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems
- communicate arguments and strategies when solving problems.

Content Descriptions

Topic 1: Exponential functions

Indices and the index laws:

- review indices (including fractional indices) and the index laws (ACMMM061)
- use radicals and convert to and from fractional indices (ACMMM062)
- understand and use scientific notation and significant figures. (ACMMM063)

Exponential functions:

- establish and use the algebraic properties of exponential functions (ACMMM064)
- recognise the qualitative features of the graph of $y = a^x \ (a > 0)$ including asymptotes, and of its translations ($y = a^x + b$ and $y = a^{x+c}$) (ACMMM065)
- identify contexts suitable for modelling by exponential functions and use them to solve practical problems (ACMMM066)
- solve equations involving exponential functions using technology, and algebraically in simple cases. (ACMMM067)

Topic 2 Arithmetic and geometric sequences and series

Arithmetic sequences:

- recognise and use the recursive definition of an arithmetic sequence: $t_{n+1} = t_n + d$ (ACMMM068)
- use the formula $t_n = t_1 + (n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature (ACMMM069)
- use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest (ACMMM070)
- establish and use the formula for the sum of the first n terms of an arithmetic sequence. (ACMMM071)

Geometric sequences:

- recognise and use the recursive definition of a geometric sequence: $t_{n+1} = rt_n$ (ACMMM072)
- use the formula $t_n = r^{n-1}t_1$ for the general term of a geometric sequence and recognise its exponential nature (ACMMM073)
- understand the limiting behaviour as $n \to \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r (ACMMM074)
- establish and use the formula $S_n = t_1 \frac{r^n - 1}{r - 1}$ for the sum of the first n terms of a geometric sequence (ACMMM075)
- use geometric sequences in contexts involving geometric growth or decay, such as compound interest. (ACMMM076)

Topic 3: Introduction to differential calculus

Rates of change:

- interpret the difference quotient $\frac{f(x+h) - f(x)}{h}$ as the average rate of change of a function f (ACMMM077)
- use the Leibniz notation δx and δy for changes or increments in the variables x and y (ACMMM078)
- use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h) - f(x)}{h}$ where $y = f(x)$ (ACMMM079)

- interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y=f(x)$. (ACMMM080)

The concept of the derivative:

- examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit (ACMMM081)
- define the derivative $f'(x)$ as $\lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$ (ACMMM082)
- use the Leibniz notation for the derivative: $\frac{dy}{dx}=\lim\limits_{\delta x\to 0}\frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx}=f'(x)$ where $y=f(x)$ (ACMMM083)
- interpret the derivative as the instantaneous rate of change (ACMMM084)
- interpret the derivative as the slope or gradient of a tangent line of the graph of $y=f(x)$. (ACMMM085)

Computation of derivatives:

- estimate numerically the value of a derivative, for simple power functions (ACMMM086)
- examine examples of variable rates of change of non-linear functions (ACMMM087)
- establish the formula $\frac{d}{dx}(x^n)=nx^{n-1}$ for positive integers n by expanding $(x+h)^n$ or by factorising $(x+h)^n-x^n$. (ACMMM088)

Properties of derivatives:

- understand the concept of the derivative as a function (ACMMM089)
- recognise and use linearity properties of the derivative (ACMMM090)
- calculate derivatives of polynomials and other linear combinations of power functions. (ACMMM091)

Applications of derivatives:

- find instantaneous rates of change (ACMMM092)
- find the slope of a tangent and the equation of the tangent (ACMMM093)
- construct and interpret position-time graphs, with velocity as the slope of the tangent (ACMMM094)
- sketch curves associated with simple polynomials; find stationary points, and local and global maxima and minima; and examine behaviour as $x\to\infty$ and $x\to-\infty$ (ACMMM095)
- solve optimisation problems arising in a variety of contexts involving simple polynomials on finite interval domains. (ACMMM096)

Anti-derivatives:

- calculate anti-derivatives of polynomial functions and apply to solving simple problems involving motion in a straight line. (ACMMM097)

Mathematical Methods

Units 1 and 2 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of functions, calculus and statistics in routine and non-routine problems in a variety of contexts • selects and applies techniques in functions, calculus and statistics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models in routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of functions, calculus and statistics in routine and non-routine problems • selects and applies techniques in functions, calculus and statistics to solve routine and non-routine problems • selects and applies mathematical and statistical models in routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of functions, calculus and statistics that apply to routine problems • selects and applies techniques in functions, calculus and statistics to solve routine problems • applies mathematical and statistical models in routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of simple functions, calculus and statistics • uses simple techniques in functions, calculus and statistics in routine problems • demonstrates familiarity mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	• demonstrates limited familiarity with concepts of simple functions, calculus and statistics • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical or statistical models • uses digital technologies for arithmetic calculations and to display limited mathematical and statistical information

Reasoning and Communication

A	B	C	D	E
• represents functions, calculus and statistics in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments, which are succinct and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents functions, calculus and statistics in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments, which are clear and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains the limitations of models used when developing solutions to routine problems	• represents functions, calculus and statistics in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems • identifies the limitations of models used when developing solutions to routine problems	• represents simple functions and distributions in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the result of calculations • identifies the limitations of simple models used	• represents limited mathematical or statistical information in a structured context • communicates simple mathematical and statistical information • identifies solutions to routine problems • describes with limited familiarity the appropriateness of the results of calculations • identifies simple models

Unit 3

Unit Description

In this unit the study of calculus continues with the derivatives of exponential and trigonometric functions and their applications, together with some differentiation techniques and applications to optimisation problems and graph sketching. It concludes with integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. In statistics, discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. This supports the development of a framework for statistical inference.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Content Descriptions

Topic 1: Further differentiation and applications

Exponential functions:

- estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$ using technology, for various values of $a > 0$ (ACMMM098)
- recognise that e is the unique number a for which the above limit is 1 (ACMMM099)
- establish and use the formula $\frac{d}{dx}\left(e^x\right) = e^x$ (ACMMM100)
- use exponential functions and their derivatives to solve practical problems. (ACMMM101)

Trigonometric functions:

- establish the formulas $\frac{d}{dx}(\sin x) = \cos x, \quad \text{and} \quad \frac{d}{dx}(\cos x) = -\sin x$ by numerical estimations of the limits and informal proofs based on geometric constructions (ACMMM102)
- use trigonometric functions and their derivatives to solve practical problems. (ACMMM103)

Differentiation rules:

- understand and use the product and quotient rules (ACMMM104)
- understand the notion of composition of functions and use the chain rule for determining the derivatives of composite functions (ACMMM105)
- apply the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x,$ $\frac{1}{x^n}$, $x \sin x,$ $e^{-x} \sin x$ and $f(ax+b)$. (ACMMM106)

The second derivative and applications of differentiation:

- use the increments formula: $\delta y \cong \frac{dy}{dx} \times \delta x$ to estimate the change in the dependent variable y resulting from changes in the independent variable x (ACMMM107)
- understand the concept of the second derivative as the rate of change of the first derivative function (ACMMM108)
- recognise acceleration as the second derivative of position with respect to time (ACMMM109)
- understand the concepts of concavity and points of inflection and their relationship with the second derivative (ACMMM110)
- understand and use the second derivative test for finding local maxima and minima (ACMMM111)
- sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection (ACMMM112)
- solve optimisation problems from a wide variety of fields using first and second derivatives. (ACMMM113)

Topic 2: Integrals

Anti-differentiation:

- recognise anti-differentiation as the reverse of differentiation (ACMMM114)
- use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals (ACMMM115)
- establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$ (ACMMM116)
- establish and use the formula $\int e^x dx = e^x + c$ (ACMMM117)

- establish and use the formulas $\int \sin x dx = -\cos x + c$ and $\int \cos x dx = \sin x + c$ (ACMMM118)
- recognise and use linearity of anti-differentiation (ACMMM119)
- determine indefinite integrals of the form $\int f(ax+b)dx$ (ACMMM120)
- identify families of curves with the same derivative function (ACMMM121)
- determine $f(x)$, given $f^{'}\left(x\right)$ and an initial condition $f(a) = b$ (ACMMM122)
- determine displacement given velocity in linear motion problems. (ACMMM123)

Definite integrals:

- examine the area problem, and use sums of the form $\sum_i f(x_i)\ \delta x_i$ to estimate the area under the curve $y = f(x)$ (ACMMM124)
- interpret the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$ (ACMMM125)
- recognise the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\ \delta x_i$ (ACMMM126)
- interpret $\int_a^b f(x)dx$ as a sum of signed areas (ACMMM127)
- recognise and use the additivity and linearity of definite integrals. (ACMMM128)

Fundamental theorem:

- understand the concept of the signed area function $F(x) = \int_a^x f(t)dt$ (ACMMM129)
- understand and use the theorem: $F^{'}(x) = \frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, and illustrate its proof geometrically (ACMMM130)
- understand the formula $\int_a^b f(x)dx = F(b) - F(a)$ and use it to calculate definite integrals. (ACMMM131)

Applications of integration:

- calculate the area under a curve (ACMMM132)
- calculate total change by integrating instantaneous or marginal rate of change (ACMMM133)
- calculate the area between curves in simple cases (ACMMM134)
- determine positions given acceleration and initial values of position and velocity. (ACMMM135)

Topic 3: Discrete random variables

General discrete random variables:

- understand the concepts of a discrete random variable and its associated probability function, and their use in modelling data (ACMMM136)
- use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable (ACMMM137)
- recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes (ACMMM138)
- examine simple examples of non-uniform discrete random variables (ACMMM139)
- recognise the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases (ACMMM140)
- recognise the variance and standard deviation of a discrete random variable as a measures of spread, and evaluate them in simple cases (ACMMM141)

- use discrete random variables and associated probabilities to solve practical problems. (ACMMM142)

Bernoulli distributions:

- use a Bernoulli random variable as a model for two-outcome situations (ACMMM143)
- identify contexts suitable for modelling by Bernoulli random variables (ACMMM144)
- recognise the mean p and variance $p(1-p)$ of the Bernoulli distribution with parameter p (ACMMM145)
- use Bernoulli random variables and associated probabilities to model data and solve practical problems. (ACMMM146)

Binomial distributions:

- understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial (ACMMM147)
- identify contexts suitable for modelling by binomial random variables (ACMMM148)
- determine and use the probabilities $\mathrm{P}(\mathrm{X}=\mathrm{r})=\binom{\mathrm{n}}{\mathrm{r}}\mathrm{p}^{\mathrm{r}}(1-\mathrm{p})^{\mathrm{n}-\mathrm{r}}$ associated with the binomial distribution with parameters n and p ; note the mean np and variance $\mathrm{np}(1-\mathrm{p})$ of a binomial distribution (ACMMM149)
- use binomial distributions and associated probabilities to solve practical problems. (ACMMM150)

Unit 4

Unit Description

The calculus in this unit deals with derivatives of logarithmic functions. In probability and statistics, continuous random variables and their applications are introduced and the normal distribution is used in a variety of contexts. The study of statistical inference in this unit is the culmination of earlier work on probability and random variables. Statistical inference is one of the most important parts of statistics, in which the goal is to estimate an unknown parameter associated with a population using a sample of data drawn from that population. In Mathematical Methods statistical inference is restricted to estimating proportions in two-outcome populations.

Access to technology to support the computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in calculus, probabilty and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Content Descriptions

Topic 1: The logarithmic function

Logarithmic functions:

- define logarithms as indices: $a^x = b$ is equivalent to $x = \log_a b$ i.e. $a^{\log_a b} = b$ (ACMMM151)
- establish and use the algebraic properties of logarithms (ACMMM152)
- recognise the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a y$ (ACMMM153)
- interpret and use logarithmic scales such as decibels in acoustics, the Richter Scale for earthquake magnitude, octaves in music, pH in chemistry (ACMMM154)
- solve equations involving indices using logarithms (ACMMM155)
- recognise the qualitative features of the graph of $y = \log_a x$ $(a > 1)$ including asymptotes, and of its translations $y = \log_a x + b$ and $y = \log_a (x + c)$ (ACMMM156)
- solve simple equations involving logarithmic functions algebraically and graphically (ACMMM157)
- identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems. (ACMMM158)

Calculus of logarithmic functions:

- define the natural logarithm $\ln x = \log_e x$ (ACMMM159)
- recognise and use the inverse relationship of the functions $y = e^x$ and $y = \ln x$ (ACMMM160)
- establish and use the formula $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (ACMMM161)
- establish and use the formula $\int \frac{1}{x} dx = \ln\ x\ + c$, for $x > 0$ (ACMMM162)
- use logarithmic functions and their derivatives to solve practical problems. (ACMMM163)

Topic 2: Continuous random variables and the normal distribution

General continuous random variables:

- use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable (ACMMM164)
- understand the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts (ACMMM165)
- recognise the expected value, variance and standard deviation of a continuous random variable and evaluate them in simple cases (ACMMM166)
- understand the effects of linear changes of scale and origin on the mean and the standard deviation. (ACMMM167)

Normal distributions:

- identify contexts such as naturally occurring variation that are suitable for modelling by normal random variables (ACMMM168)
- recognise features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution (ACMMM169)
- calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve

practical problems. (ACMMM170)

Topic 3: Interval estimates for proportions

Random sampling:

- understand the concept of a random sample (ACMMM171)
- discuss sources of bias in samples, and procedures to ensure randomness (ACMMM172)
- use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli. (ACMMM173)

Sample proportions:

- understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{(p(1-p)/n}$ of the sample proportion $\hat{p}$ (ACMMM174)
- examine the approximate normality of the distribution of $\hat{p}$ for large samples (ACMMM175)
- simulate repeated random sampling, for a variety of values of *p* and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{(\hat{p}(1-\hat{p})/n}}$ where the closeness of the approximation depends on both *n* and *p*. (ACMMM176)

Confidence intervals for proportions:

- the concept of an interval estimate for a parameter associated with a random variable (ACMMM177)
- use the approximate confidence interval $\left(\hat{p}-z\sqrt{(\hat{p}(1-\hat{p})/n},\ \hat{p}+z\sqrt{(\hat{p}(1-\hat{p})/n}\right)$, as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution (ACMMM178)
- define the approximate margin of error $E=z\sqrt{(\hat{p}(1-\hat{p})/n}$ and understand the trade-off between margin of error and level of confidence (ACMMM179)
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain p. (ACMMM180)

Units 3 and 4 Achievement Standards

Concepts and techniques

A	B	C	D	E
• demonstrates knowledge of concepts of functions, integration and distributions in routine and non-routine problems in a variety of contexts • selects and applies techniques in functions, integration and distributions to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models in routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of functions, integration and distributions in routine and non-routine problems • selects and applies techniques in functions, integration and distributions to solve routine and non-routine problems • selects and applies mathematical and statistical models in routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information and to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of functions, integration and distributions that apply to routine problems • selects and applies techniques in functions, integration and distributions to solve routine problems • applies mathematical and statistical models in routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of simple functions, integration and distributions • uses simple techniques in functions, integration and distributions in routine problems • demonstrates familiarity with mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	• demonstrates limited familiarity with concepts of simple functions, integration and distributions • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical or statistical models • uses digital technologies for arithmetic calculations and to display limited mathematical and statistical information

Reasoning and Communication

A	B	C	D	E
• represents functions, integration and distributions in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments, which are succinct and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents functions, integration and distributions in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments, which are clear and reasoned, using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains the limitations of models used when developing solutions to routine problems	• represents functions, integration and distributions in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems • identifies thelimitations of models used when developing solutions to routine problems	• represents simple functions and distributions in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the result of calculations • identifies limitations of simple models used	• represents limited mathematical or statistical information in a structured context • communicates simple mathematical and statistical information • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

Mathematical Methods Glossary

Additivity property of definite integrals

The additivity property of definite integrals refers to 'addition of intervals of integration':

$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ for any numbers a, b and c and any function $f(x)$.

Algebraic properties of exponential functions

The algebraic properties of exponential functions are the index laws: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, for any real numbers $x,\ y,\ \text{and}\ a$, with $a > 0$

Algebraic properties of logarithms

The algebraic properties of logarithms are the rules: $\log_a(xy) = \log_a x + \log_a y$,$\log_a \frac{1}{x} = -\log_a x$, and $\log_a 1 = 0$, for any positive real numbers $x,\ y\ \text{and}\ a$

Antidifferentiation

An anti-derivative, primitive or indefinite integral of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'\left(x\right) = f\left(x\right)$.

The process of solving for anti-derivatives is called anti-differentiation.

Anti-derivatives are not unique. If $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number. We write $\int f(x)dx = F(x) + c$ to denote the set of all anti-derivatives of $f(x)$. The number c is called the constant of integration. For example, since $\frac{d}{dx}\left(x^3\right) = 3x^2$, we can write $\int 3x^2\ dx = x^3 + c$

Arithmetic sequence

An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence

2, 5, 8, 11, 14, 17, …

is an arithmetic sequence with common difference 3.

If the initial term of an arithmetic sequence is a and the common difference of successive members is d, then the nth term tn, of the sequence, is given by:

$t_n = a + (n - 1)d$ for $n = 1$

A recursive definition is

$t_1 = a$, $t_{n+1} = t_{n + d}$ where d is the common difference and n = 1.

Asymptote

A line is an asymptote to a curve if the distance between the line and the curve approaches zero as they 'tend to infinity'. For example, the line with equation $x = \pi/2$ is a vertical asymptote to the graph of $y = \tan x$, and the line with equation $y = 0$ is a horizontal asymptote to the graph of $y = 1/x$.

Bernoulli random variable

A Bernoulli random variable has two possible values, namely 0 and 1. The parameter associated with such a random variable is the probability p of obtaining a 1.

Bernoulli trial

A Bernoulli trial is a chance experiment with possible outcomes, typically labeled 'success' and failure'.

Binomial distribution

The expansion $(x+y)^n = x^n + \binom{n}{1} x^{n-1}y + \cdots + \binom{n}{r} x^{n-r}y^r + \cdots + y^n$ is known as the binomial theorem. The numbers $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n\times(n-1)\times\cdots\times(n-r+1)}{r\times(r-1)\times\cdots\times 2\times 1}$ are called binomial coefficients.

Central limit theorem

There are various forms of the Central limit theorem, a result of fundamental importance in statistics. For the purposes of this course, it can be expressed as follows:

"If X is the mean of n independent values of random variable X which has a finite mean μ and a finite standard deviation σ, then as $n \to \infty$ the distribution of $\frac{X-\mu}{\sigma/\sqrt{n}}$ approaches the standard normal distribution."

In the special case where X is a Bernoulli random variable with parameter p, X is the sample proportion $\hat{p}, \mu = p$ and $\sigma = \sqrt{p(1-p)}$. In this case the Central limit theorem is a statement that as $n \to \infty$ the distribution of $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$ approaches the standard normal distribution.

Chain rule

The chain rule relates the derivative of the composite of two functions to the functions and their derivatives.

If $h(x) = f \circ g(x)$ then $(f \circ g)^{'}(x) = f^{'}(g(x))g^{'}(x)$,

and in Leibniz notation: $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$

Circular measure

is the measurement of angle size in radians.

Completing the square

The quadratic expression $ax^2 + bx + c$ can be rewritten as $a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$. Rewriting it in this way is called completing the square.

Composition of functions

If $\mathrm{y} = g(x)$ and $z = f(y)$ for functions f and g, then z is a composite function of x. We write $z = f \circ g(x) = f(g(x))$. For example, $z = \sqrt{x^2+3}$ expresses z as a composite of the functions $\mathrm{f}(\mathrm{y}) = \sqrt{\mathrm{y}}$ and $\mathrm{g}(\mathrm{x}) = \mathrm{x}^2 + 3$

Concave up and concave down

A graph of $y = f(x)$ is concave up at a point P if points on the graph near P lie above the tangent at P. The graph is concave down at P if points on the graph near P lie below the tangent at P.

Conditional probability

The probability that an event A occurs can change if it becomes known that another event B occurs. The new probability is known as a conditional probability and is written as $P(A|B)$. If B has occurred, the sample space is reduced by discarding all outcomes that are not in the event B. The new sample space, called the reduced sample space, is B. The conditional probability of event A is given by $P\left(A\middle|B\right) = \frac{P(A\cap B)}{P(B)}$.

Discriminant

The discriminant of the quadratic expression $ax^2 + bx + c$ is the quantity $b^2 - 4ac$

Effect of linear change

The effects of linear changes of scale and origin on the mean and variance of a random variable are summarized as follows:

If X is a random variable and $Y = aX + b$, where a and b are constants, then

$E(Y) = aE(X) + b$ and $Var(Y) = a^2 Var(X)$

Euler's number

Euler's number$e$ is an irrational number whose decimal expansion begins

$e = 2.7182818284590452353602874713527\cdots$

It is the base of the natural logarithms, and can be defined in various ways including:

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ and $e = \lim\limits_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$.

Expected value

The expected value$E(X)$ of a random variable X is a measure of the central tendency of its distribution.

If X is discrete, $E(X) = \sum_i p_i x_i$, where the x_i are the possible values of X and

$p_i = P(X = x_i)$.

If X is continuous, $E(x) = \int_{-\infty}^{\infty} xp(x)dx$, where $p(x)$ is the probability density function of X

Function

A function$f$ is a rule that associates with each element x in a set S a unique element $f(x)$ in a set T. We write $x \longmapsto f(x)$ to indicate the mapping of x to $f(x)$. The set S is called the domain of f and the set T is called the codomain. The subset of T consisting of all the elements $f(x) : x \in S$ is called the range of f. If we write $y = f(x)$ we say that x is the independent variable and y is the dependent variable.

Geometric sequence

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the common ratio. For example, the sequence

3, 6, 12, 24, ...

is a geometric sequence with common ratio 2. Similarly the sequence

40, 20, 10, 5, 2.5, …

is a geometric sequence with common ratio $\frac{1}{2}$.

If the initial term of a geometric sequence is a and the common ratio of successive members is r, then the nth term tn, of the sequence, is given by:

$t_n = ar^{n-1}$ for $n = 1$

A recursive definition is

$t_1 = a$, $t_{n+1} = rt_n$ for n = 1 and where r is the constant ratio

Gradient (Slope)

The gradient of the straight line passing through points (x_1, y_1) and (x_2, y_2) is the ratio $\frac{y_2-y_1}{x_2-x_1}$. Slope is a synonym for gradient.

Graph of a function

The graph of a function f is the set of all points (x, y) in Cartesian plane where x is in the domain of f and $y = f(x)$

Independent events

Two events are independent if knowing that one occurs tells us nothing about the other. The concept can be defined formally using probabilities in various ways: events A and B are independent if $P(A \cap B) = P(A)P(B)$, if $P(A|B) = P(A)$ or if $P(B) = P(B|A)$. For events A and B with non-zero probabilities, any one of these equations implies any other.

Index laws

The index laws are the rules: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, and $(ab)^x = a^x b^x$, for any real numbers $x,\ y,\ a \text{ and } b$, with $a > 0 \text{ and } b > 0$

Length of an arc

The length of an arc in a circle is given by $l = r\theta$, where l is the arc length, r is the radius and θ is the angle subtended at the centre, measured in radians. This is simply a rearrangement of the formula defining the radian measure of an angle.

Level of confidence

The level of confidence associated with a confidence interval for an unknown population parameter is the probability that a random confidence interval will contain the parameter.

linearity property of the derivative

The linearity property of the derivative is summarized by the equations:

$\frac{d}{dx}(ky) = k\frac{dy}{dx}$ for any constant k

and $\frac{d}{dx}(y_1 + y_2) = \frac{dy_1}{dx} + \frac{dy_2}{dx}$

Local and global maximum and minimum

A stationarypoint on the graph $y = f(x)$ of a differentiable function is a point where $f'(x) = 0$.

We say that $f(x_0)$ is a local maximum of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a global maximum of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x in the domain of f.

We say that $f(x_0)$ is a local minimum of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a global minimum of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x in the domain of f.

Margin of error

The margin of error of a confidence interval of the form $f - E < p < f + E$ is E, the half-width of the confidence interval. It is the maximum difference between f and p if p is actually in the confidence interval.

Mean of a random variable

The mean of a random variable is another name for its expected value.

identity matrix The variance$Var(X)$ of a random variable X is a measure of the 'spread' of its distribution.

If X is discrete, $Var(X) = \sum_i p_i(x_i - \mu)^2$, where $\mu = E(X)$ is the expected value.

If X is continuous, $Var(X) = \int_{-\infty}^{\infty} (x - \mu)^2 p(x)dx$

Mutually exclusive

Two events are mutually exclusive if there is no outcome in which both events occur.

Partial sum of an arithmetic sequence (Arithmetic series)

The partial sum Sn of the first n terms of an arithmetic sequence with first term a and common difference d.

a, *a* + *d*, *a* + 2*d*,……, *a* + (*n* – 1)*d*,…

is

Sn = $\frac{n}{2}$ (a + tn) = $\frac{n}{2}$ (2a + (n – 1)d) where tn is the nth term of the sequence.

The partial sums form a sequence with Sn+1 = Sn + tn and S1 = t1

Partial sums of a geometric sequence (Geometric series)

The partial sum Sn of the first n terms of a geometric sequence with first term a and common ratio r,

$a, ar, ar^2, \ldots\ldots, ar^{n-1}, \ldots$

is

$S_n = \frac{a(r^n - 1)}{r-1}$, $r \neq 1$.

The partial sums form a sequence with Sn+1 = Sn + tn and S1 = t1.

Partial sums of a sequence (Series)

The sequence of partial sums of a sequence t1,…, ts, … is defined by

$S_n = t_1 + \ldots + t_n$

Pascal's triangle

Pascal's triangle is a triangular arrangement of binomial coefficients. The n^{th} row consists of the binomial coefficients $\binom{n}{r}$, for $0 \le r \le n$, each interior entry is the sum of the two entries above it, and sum of the entries in the n^{th} row is 2^n

Period of a function

The period of a function $f(x)$ is the smallest positive number p with the property that $f(x+p) = f(x)$ for all x. The functions $\sin x$ and $\cos x$ both have period 2π and $\tan x$ has period π

Point and interval estimates

In statistics estimation is the use of information derived from a sample to produce an estimate of an unknown probability or population parameter. If the estimate is a single number, this number is called a point estimate. An interval estimate is an interval derived from the sample that, in some sense, is likely to contain the parameter.

A simple example of a point estimate of the probability p of an event is the relative frequency f of the event in a large number of Bernoulli trials. An example of an interval estimate for p is a confidence interval centred on the relative frequency f.

Point of inflection

A point P on the graph of $y = f(x)$ is a point of inflection if the concavity changes at P, i.e. points near P on one side of P lie above the tangent at P and points near P on the other side of P lie below the tangent at P

Probability density function

The probability density function of a continuous random variable is a function that describes the relative likelihood that the random variable takes a particular value. Formally, if $p(x)$ is the probability density of the continuous random variable X, then the probability that X takes a value in some interval $[a, b]$ is given by $\int_a^b p(x)\, dx$.

Probability distribution

The probability distribution of a discrete random variable is the set of probabilities for each of its possible values.

Product rule

The product rule relates the derivative of the product of two functions to the functions and their derivatives.

If $h(x) = f(x)g(x)$ then $h^{'}(x) = f(x)g^{'}(x) + f^{'}(x)g\left(x\right)$,

and in Leibniz notation: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

Quadratic formula

If $ax^2 + bx + c = 0$ with $a \neq 0$, then $x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$. This formula for the roots is called the quadratic formula.

Quantile

A quantilet_α for a continuous random variable X is defined by $P(X > t_\alpha) = \alpha$, where $0 < \alpha < 1.$

The median$m$ of X is the quantile corresponding to $\alpha = 0.5$: $P(X > m) = 0.5$

Quotient rule

The quotient rule relates the derivative of the quotient of two functions to the functions and their derivatives

If $h(x) = \frac{f\left(x\right)}{g\left(x\right)}$ then $h'(x) = \frac{g\left(x\right)f^{'}\left(x\right) - f\left(x\right)g^{'}\left(x\right)}{g\left(x\right)^2}$

and in Leibniz notation: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Radian measure

The radian measureθ of an angle in a sector of a circle is defined by , where is the radius and is the arc length. Thus an angle whose degree measure is has radian measure .

Random variable

A random variable is a numerical quantity whose value depends on the outcome of a chance experiment. Typical examples are the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin, and the number of tonnes of wheat produced in Australia in a year.

A discrete random variable is one whose possible values are the counting numbers , or form a finite set, as in the first two examples.

A continuous random variable is one whose set of possible values are all of the real numbers in some interval.

Relative frequency

If an event occurs times when a chance experiment is repeated times, the relative frequency of is .

Unit 2

Secant

A secant of the graph of a function is the straight line passing through two points on the graph. The line segment between the two points is called a chord.

Second derivative test

According to the second derivative test, if then is a local maximum of if and is a local minimum if

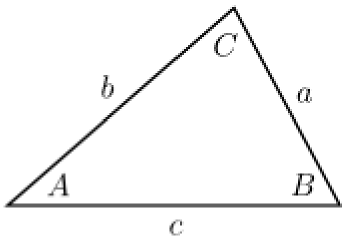
Sine and cosine functions

In the unit circle definition of cosine and sine, are the coordinates of the point on the unit circle corresponding to the angle

Sine rule and cosine rule

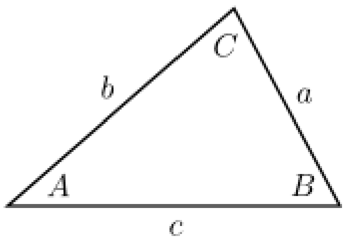
The lengths of the sides of a triangle are related to the sines of its angles by the equations

This is known as the sine rule.



The lengths of the sides of a triangle are related to the cosine of one of its angles by the equation

This is known as the cosine rule.



Standard deviation of a random variable

The standard deviation of a random variable is the square root of its variance.

Tangent line

The tangent line (or simply the tangent) to a curve at a given point can be described intuitively as the straight line that "just touches" the curve at that point. At the point where the tangent touches the curve, the curve has “the same

direction” as the tangent line.”. In this sense it is the best straight-line approximation to the curve at the point .

The fundamental theorem of calculus

The fundamental theorem of calculus relates differentiation and definite integrals. It has two forms:

The linearity property of anti-differentiation

The linearity property of anti-differentiation is summarized by the equations:

for any constant and

for any two functions

Similar equations describe the linearity property of definite integrals:

for any constant and

for any two functions

Triangular continuous random variable

A triangular continuous random variable is one whose probability density function has a graph with the shape of a triangle.

Uniform continuous random variable

A uniform continuous random variable is one whose probability density function has constant value on the range of possible values of . If the range of possible values is the interval then if and otherwise.

Uniform discrete random variable

A uniform discrete random variable is one whose possible values have equal probability of occurrence. If there are possible values, the probability of occurrence of any one of them is .

Vertical line test

A relation between two real variables and is a function and for some function if and only if each vertical line, i.e. each line parallel to the axis, intersects the graph of the relation in at most one point. This test to determine whether a relation is, in fact, a function is known as the vertical line test.

Typesetting math: 94%

Glossary

Abstract

Abstract scenario: a scenario for which there is no concrete referent provided.

Account

Account for: provide reasons for (something).

Give an account of: report or describe an event or experience.

Taking into account: considering other information or aspects.

Analyse

Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences.

Apply

Use, utilise or employ in a particular situation.

Assess

Determine the value, significance or extent of (something).

Coherent

Orderly, logical, and internally consistent relation of parts.

Communicates

Conveys knowledge and/or understandings to others.

Compare

Estimate, measure or note how things are similar or dissimilar.

Complex

Consisting of multiple interconnected parts or factors.

Considered

Formed after careful thought.

Critically analyse

Examine the component parts of an issue or information, for example the premise of an argument and its plausibility, illogical reasoning or faulty conclusions

Critically evaluate

Evaluation of an issue or information that includes considering important factors and available evidence in making critical judgement that can be justified.

Deduce

Arrive at a conclusion by reasoning.

Demonstrate

Give a practical exhibition as an explanation.

Describe

Give an account of characteristics or features.

Design

Plan and evaluate the construction of a product or process.

Develop

In history: to construct, elaborate or expand.

In English: begin to build an opinion or idea.

Discuss

Talk or write about a topic, taking into account different issues and ideas.

Distinguish

Recognise point/s of difference.

Evaluate

Provide a detailed examination and substantiated judgement concerning the merit, significance or value of something.

In mathematics: calculate the value of a function at a particular value of its independent variables.

Explain

Provide additional information that demonstrates understanding of reasoning and/or application.

Familiar

Previously encountered in prior learning activities.

Identify

Establish or indicate who or what someone or something is.

Integrate

Combine elements.

Investigate

Plan, collect and interpret data/information and draw conclusions about.

Justify

Show how an argument or conclusion is right or reasonable.

Locate

Identify where something is found.

Manipulate

Adapt or change.

Non-routine

Non-routine problems: Problems solved using procedures not previously encountered in prior learning activities.

Reasonableness

Reasonableness of conclusions or judgements: the extent to which a conclusion or judgement is sound and makes sense

Reasoned

Reasoned argument/conclusion: one that is sound, well-grounded, considered and thought out.

Recognise

Be aware of or acknowledge.

Relate

Tell or report about happenings, events or circumstances.

Represent

Use words, images, symbols or signs to convey meaning.

Reproduce

Copy or make close imitation.

Responding

In English: When students listen to, read or view texts they interact with those texts to make meaning. Responding involves students identifying, selecting, describing, comprehending, imagining, interpreting, analysing and evaluating.

Routine problems

Routine problems: Problems solved using procedures encountered in prior learning activities.

Select

Choose in preference to another or others.

Sequence

Arrange in order.

Solve

Work out a correct solution to a problem.

Structured

Arranged in a given organised sequence.

In Mathematics: When students provide a structured solution, the solution follows an organised sequence provided by a third party.

Substantiate

Establish proof using evidence.

Succinct

Written briefly and clearly expressed.

Sustained

Consistency maintained throughout.

Synthesise

Combine elements (information/ideas/components) into a coherent whole.

Understand

Perceive what is meant, grasp an idea, and to be thoroughly familiar with.

Unfamiliar

Not previously encountered in prior learning activities.