The Australian Curriculum

Subjects	General Mathematics
Units	Unit 1, Unit 2, Unit 3 and Unit 4
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AUSTRALIAN CURRICULUM, ASSESSMENT AND REPORTING AUTHORITY

Table of Contents

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General Mathematics

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General Mathematics

Rationale and Aims

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe many aspects of the world in the twenty-first century. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve practical problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

General Mathematics is designed for those students who want to extend their mathematical skills beyond Year 10 level but whose future studies or employment pathways do not require knowledge of calculus. The subject is designed for students who have a wide range of educational and employment aspirations, including continuing their studies at university or TAFE.

The proficiency strands of the F-10 curriculum – Understanding, Fluency, Problem solving and Reasoning – are still relevant and are inherent in all aspects of this subject. Each of these proficiencies is essential, and all are mutually reinforcing. Fluency, for example, might include learning to perform routine calculations efficiently and accurately, or being able to recognise quickly from a problem description the appropriate mathematical process or model to apply. Understanding, furthermore, that a single mathematical process can be used in seemingly different situations, helps students to see the connections between different areas of study and encourages the transfer of learning. This is an important part of learning the art of mathematical problem solving. In performing such analyses, reasoning is required at each decision-making step and in drawing appropriate conclusions. Presenting the analysis in a logical and clear manner to explain the reasoning used is also an integral part of the learning process.

Throughout the subject there is also an emphasis on the use and application of digital technologies.

Aims

General Mathematics aims to develop students':

- understanding of concepts and techniques drawn from the topic areas of number and algebra, geometry and trigonometry, graphs and networks, and statistics
- ability to solve applied problems using concepts and techniques drawn from the topic areas of number and algebra, geometry and trigonometry, graphs and networks, and statistics
- reasoning and interpretive skills in mathematical and statistical contexts
- capacity to communicate the results of a mathematical or statistical problem-solving activity in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Organisation

Overview of senior secondary Australian Curriculum

ACARA has developed senior secondary Australian Curriculum for English, Mathematics, Science and History according to a set of design specifications. The ACARA Board approved these specifications following consultation with state and territory curriculum, assessment and certification authorities.

Each senior secondary Australian Curriculum specifies content and achievement standards for a senior secondary subject. Content refers to the knowledge, understanding and skills to be taught and learned within a given subject. Achievement standards refer to descriptions of the quality of learning (the depth of understanding, extent of knowledge and sophistication of skill) demonstrated by students who have studied the content for the subject.

The senior secondary Australian Curriculum for each subject has been organised into four units. The last two units are cognitively more challenging than the first two units. Each unit is designed to be taught in about half a school year of senior secondary studies (approximately 50–60 hours duration including assessment). However, the senior secondary units have also been designed so that they may be studied singly, in pairs (that is, over a year), or as four units over two years. State and territory curriculum, assessment and certification authorities are responsible for the structure and organisation of their senior secondary courses and will determine how they will integrate the Australian Curriculum content and achievement standards into courses. They will also provide any advice on entry and exit points, in line with their curriculum, assessment and certification requirements.

States and territories, through their respective curriculum, assessment and certification authorities, will continue to be responsible for implementation of the senior secondary curriculum, including assessment, certification and the attendant quality assurance mechanisms. Each of these authorities acts in accordance with its respective legislation and the policy framework of its state government and board. They will determine the assessment and certification specifications for their courses that use the Australian Curriculum content and achievement standards, as well as any additional information, guidelines and rules to satisfy local requirements.

This curriculum should not, therefore, be read as proposed courses of study. Rather, it presents content and achievement standards for integration into state and territory courses.

Senior secondary Mathematics subjects

The Senior Secondary Australian Curriculum: Mathematics consists of four subjects in mathematics, with each subject organised into four units. The subjects are differentiated, each focusing on a pathway that will meet the learning needs of a particular group of senior secondary students.

Essential Mathematics focuses on using mathematics effectively, efficiently and critically to make informed decisions. It provides students with the mathematical knowledge, skills and understanding to solve problems in real contexts for a range of workplace, personal, further learning and community settings. It also provides the opportunity for students to prepare for post-school options of employment or further training.

General Mathematics focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve analysing univariate and bivariate data, including time series data.

Mathematical Methods focuses on the use of calculus and statistical analysis. The study of calculus provides a basis for understanding rates of change in the physical world, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics develops students' ability to describe and analyse phenomena that involve uncertainty and variation.

Specialist Mathematics provides opportunities, beyond those in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends students' understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. It is the only mathematics subject that cannot be taken as a stand-alone subject.

Structure of General Mathematics

General Mathematics is organised into four units. The topics in each unit broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic, geometric and statistical thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction.

Unit 1	Unit 2	Unit 3	Unit 4
Consumer arithmetic Algebra and matrices Shape and measurement	Univariate data analysis and the statistical investigation process Applications of trigonometry Linear equations and their graphs	Bivariate data analysis Growth and decay in sequences Graphs and networks	Time series analysis Loans, investments and annuities Networks and decision mathematics

Units

Unit 1 has three topics: 'Consumer arithmetic', 'Algebra and matrices', and 'Shape and measurement'. 'Consumer arithmetic' reviews the concepts of rate and percentage change in the context of earning and managing money, and provides fertile ground for the use of spreadsheets. 'Algebra and matrices' continues the F-10 study of algebra and introduces the new topic of matrices. 'Shape and measurement' extends the knowledge and skills students developed in the F-10 curriculum with the concept of similarity and associated calculations involving simple and compound geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Unit 2 has three topics: 'Univariate data analysis and the statistical investigation process', 'Linear equations and their graphs', and 'Applications of trigonometry'. 'Univariate data analysis and the statistical investigation process' develops students' ability to organise and summarise univariate data in the context of conducting a statistical investigation. . 'Applications of trigonometry' extends students' knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression, and bearings in navigation 'Linear equations and their graphs' uses linear equations and straight-line graphs, as well as linear-piecewise and step graphs, to model and analyse practical situations

Unit 3 has three topics: ‘Bivariate data analysis’, ‘Growth and decay in sequences’, and ‘Graphs and networks’. ‘Bivariate data analysis’ introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including using the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process. ‘Growth and decay in sequences’ employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4. ‘Graphs and networks’ introduces students to the language of graphs and the way in which graphs, represented as a collection of points and interconnecting lines, can be used to analyse everyday situations such as a rail or social network.

Unit 4 has three topics: ‘Time series analysis’, ‘Loans, investments and annuities’, and ‘Networks and decision mathematics’. ‘Time series analysis’ continues students’ study of statistics by introducing them to the concepts and techniques of time series analysis. The content is to be taught within the framework of the statistical investigation process. ‘Loans and investments’ aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments. ‘Networks and decision mathematics’ uses networks to model and aid decision making in practical situations.

Organisation of achievement standards

The achievement standards in Mathematics have been organised into two dimensions: ‘Concepts and Techniques’ and ‘Reasoning and Communication’. These two dimensions reflect students’ understanding and skills in the study of mathematics.

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Senior secondary achievement standards have been written for each Australian Curriculum senior secondary subject. The achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following the completion of study of senior secondary Australian Curriculum content for a pair of units. They are broad statements of understanding and skills that are best read and understood in conjunction with the relevant unit content. They are structured to reflect key dimensions of the content of the relevant learning area. They will be eventually accompanied by illustrative and annotated samples of student work/ performance/ responses.

The achievement standards will be refined empirically through an analysis of samples of student work and responses to assessment tasks: they cannot be maintained a priori without reference to actual student performance. Inferences can be drawn about the quality of student learning on the basis of observable differences in the extent, complexity, sophistication and generality of the understanding and skills typically demonstrated by students in response to well-designed assessment activities and tasks.

In the short term, achievement standards will inform assessment processes used by curriculum, assessment and certifying authorities for course offerings based on senior secondary Australian Curriculum content.

ACARA has made reference to a common syntax (as a guide, not a rule) in constructing the achievement standards across the learning areas. The common syntax that has guided development is as follows:

- Given a specified context (as described in the curriculum content)
- With a defined level of consistency/accuracy (the assumption that each level describes what the student does well, competently, independently, consistently)
- Students perform a specified action (described through a verb)
- In relation to what is valued in the curriculum (specified as the object or subject)
- With a defined degree of sophistication, difficulty, complexity (described as an indication of quality)

Terms such as ‘analyse’ and ‘describe’ have been used to specify particular action but these can have everyday meanings that are quite general. ACARA has therefore associated these terms with specific meanings that are defined in the senior secondary achievement standards glossary and used precisely and consistently across subject areas.

Role of technology

It is assumed that students will be taught the Senior Secondary Australian Curriculum: Mathematics subjects with an extensive range of technological applications and techniques. If appropriately used, these have the potential to enhance the teaching and learning of mathematics. However, students also need to continue to develop skills that do not depend on technology. The ability to choose when and when not to use some form of technology, and the ability to work flexibly with technology, are important skills in these subjects.

Links to Foundation to Year 10

The General Mathematics subject provides students with a breadth of mathematical and statistical experience that encompasses and builds on all three strands of the F-10 curriculum.

Representation of General capabilities

The seven general capabilities of *Literacy, Numeracy, Information and Communication Technology (ICT) capability, Critical and creative thinking, Personal and social capability, Ethical understanding*, and *Intercultural understanding* are identified where they offer opportunities to add depth and richness to student learning. Teachers will find opportunities to incorporate explicit teaching of the capabilities depending on their choice of learning activities.

Literacy in mathematics

In the senior years, literacy skills and strategies enable students to express, interpret and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their abilities to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in mathematics

The students who undertake this subject will develop their numeracy skills at a more sophisticated level than in Foundation to Year 10. This subject contains financial applications of mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age.

ICT in mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, using data, addressing problems, and operating systems in authentic situations.

Critical and creative thinking in mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions do not match, it is due to a flaw in the theory or in the method of applying the theory to make predictions, or both. They revise, or reapply, their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and in making accurate predictions.

Personal and social capability in mathematics

In the senior years students develop personal and social competence in mathematics by setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision making.

The elements of personal and social competence relevant to mathematics mainly include the application of mathematical skills for decision making, life-long learning, citizenship and self-management. As part of their mathematical explorations and investigations, students work collaboratively in teams, as well as independently.

Ethical understanding in mathematics

In the senior years students develop ethical understanding in mathematics through decision making connected with ethical dilemmas that arise when engaged in mathematical calculation, the dissemination of results, and the social responsibility associated with teamwork and attribution of input.

The areas relevant to mathematics include issues associated with ethical decision making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and the examined ethical behaviour. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural understanding in mathematics

Students understand mathematics as a socially constructed body of knowledge that uses universal symbols but has its origins in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number, and that diverse cultural spatial abilities and understandings are shaped by a person’s environment and language.

Representation of Cross-curriculum priorities

The senior secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander peoples and their central place in contemporary Australian society and culture. Through the study of mathematics within relevant contexts, students are given opportunities to develop their understanding and appreciation of the diversity of cultures and histories of Aboriginal and Torres Strait Islander peoples and their contribution to Australian society.

There are strong social, cultural and economic reasons for Australian students to engage with Asia and with the contribution of Asian Australians to our society and heritage. It is through the study of mathematics in an Asian context that a creative and forward-looking Australia can truly engage with our place in the region. By analysing relevant data, students have opportunities to develop an understanding of the diversity of Asia’s peoples, environments, and traditional and contemporary cultures.

Each of the senior mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Teachers are therefore encouraged to select contexts for discussion that are connected with sustainability. Through the analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world views.

Unit 1

Unit Description

This unit has three topics: ‘Consumer arithmetic’, ‘Algebra and matrices’, and ‘Shape and measurement’.

‘Consumer arithmetic’ reviews the concepts of rate and percentage change in the context of earning and managing money, and provides a fertile ground for the use of spreadsheets.

‘Algebra and matrices’ continues the F-10 study of algebra and introduces the new topic of matrices.

‘Shape and measurement’ builds on and extends the knowledge and skills students developed in the F-10 curriculum with the concept of similarity and associated calculations involving simple and compound geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Classroom access to the technology necessary to support the computational aspects of the topics in this unit is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques introduced in consumer arithmetic, algebra and matrices, and shape and measurement
- apply reasoning skills and solve practical problems arising in consumer arithmetic, algebra and matrices, and shape and measurement
- communicate their arguments and strategies, when solving problems, using appropriate mathematical language
- interpret mathematical information, and ascertain the reasonableness of their solutions to problems
- choose and use technology appropriately and efficiently.

Content Descriptions

Topic 1: Consumer arithmetic

Applications of rates and percentages:

- review rates and percentages (ACMGM001)
- calculate weekly or monthly wage from an annual salary, wages from an hourly rate including situations involving overtime and other allowances and earnings based on commission or piecework (ACMGM002)
- calculate payments based on government allowances and pensions (ACMGM003)
- prepare a personal budget for a given income taking into account fixed and discretionary spending (ACMGM004)
- compare prices and values using the unit cost method (ACMGM005)
- apply percentage increase or decrease in various contexts; for example, determining the impact of inflation on costs and wages over time, calculating percentage mark-ups and discounts, calculating GST, calculating profit or loss in absolute and percentage terms, and calculating simple and compound interest (ACMGM006)
- use currency exchange rates to determine the cost in Australian dollars of purchasing a given amount of a foreign currency, such as US$1500, or the value of a given amount of foreign currency when converted to Australian dollars, such as the value of €2050 in Australian dollars (ACMGM007)
- calculate the dividend paid on a portfolio of shares, given the percentage dividend or dividend paid per share, for each share; and compare share values by calculating a price-to-earnings ratio. (ACMGM008)

Use of spreadsheets:

- use a spreadsheet to display examples of the above computations when multiple or repeated computations are required; for example, preparing a wage-sheet displaying the weekly earnings of workers in a fast food store where hours of employment and hourly rates of pay may differ, preparing a budget, or investigating the potential cost of owning and operating a car over a year. (ACMGM009)

Topic 2: Algebra and matrices

Linear and non-linear expressions:

- substitute numerical values into linear algebraic and simple non-linear algebraic expressions, and evaluate (ACMGM010)
- find the value of the subject of the formula, given the values of the other pronumerals in the formula (ACMGM011)
- use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities; for example, a table displaying the body mass index (BMI) of people of different weights and heights. (ACMGM012)

Matrices and matrix arithmetic:

- use matrices for storing and displaying information that can be presented in rows and columns; for example, databases, links in social or road networks (ACMGM013)
- recognise different types of matrices (row, column, square, zero, identity) and determine their size (ACMGM014)
- perform matrix addition, subtraction, multiplication by a scalar, and matrix multiplication, including determining the power of a matrix using technology with matrix arithmetic capabilities when appropriate (ACMGM015)
- use matrices, including matrix products and powers of matrices, to model and solve problems; for example, costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can

communicate with each other via a third person. (ACMGM016)

Topic 3: Shape and measurement

Pythagoras' Theorem:

- review Pythagoras' Theorem and use it to solve practical problems in two dimensions and for simple applications in three dimensions. (ACMGM017)

Mensuration:

- solve practical problems requiring the calculation of perimeters and areas of circles, sectors of circles, triangles, rectangles, parallelograms and composites (ACMGM018)
- calculate the volumes of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the volume of water contained in a swimming pool (ACMGM019)
- calculate the surface areas of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the surface area of a cylindrical food container. (ACMGM020)

Similar figures and scale factors:

- review the conditions for similarity of two-dimensional figures including similar triangles (ACMGM021)
- use the scale factor for two similar figures to solve linear scaling problems (ACMGM022)
- obtain measurements from scale drawings, such as maps or building plans, to solve problems (ACMGM023)
- obtain a scale factor and use it to solve scaling problems involving the calculation of the areas of similar figures (ACMGM024)
- obtain a scale factor and use it to solve scaling problems involving the calculation of surface areas and volumes of similar solids. (ACMGM025)

Unit 2

Unit Description

This unit has three topics: 'Univariate data analysis and the statistical investigation process', 'Linear equations and their graphs'; and 'Applications of trigonometry'.

' Univariate data analysis and the statistical investigation process' develops students' ability to organise and summarise univariate data in the context of conducting a statistical investigation.

'Linear equations and their graphs' uses linear equations and straight-line graphs, as well as linear-piecewise and step graphs, to model and analyse practical situations.

'Applications of trigonometry' extends students' knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression and bearings in navigation.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in univariate data analysis and the statistical investigation process, linear equations and their graphs, and applications of trigonometry
- apply reasoning skills and solve practical problems in univariate data analysis and the statistical investigation process, linear equations and their graphs, and the applications of trigonometry
- implement the statistical investigation process in contexts requiring the analysis of univariate data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Content Descriptions

Topic 1: Univariate data analysis and the statistical investigation process

The statistical investigation process:

- review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results. (ACMGM026)

Making sense of data relating to a single statistical variable:

- classify a categorical variable as ordinal, such as income level (high, medium, low), or nominal, such as place of birth (Australia, overseas), and use tables and bar charts to organise and display the data (ACMGM027)
- classify a numerical variable as discrete, such as the number of rooms in a house, or continuous, such as the temperature in degrees Celsius (ACMGM028)
- with the aid of an appropriate graphical display (chosen from dot plot, stem plot, bar chart or histogram), describe the distribution of a numerical dataset in terms of modality (uni or multimodal), shape (symmetric versus positively or negatively skewed), location and spread and outliers, and interpret this information in the context of the data (ACMGM029)
- determine the mean and standard deviation of a dataset and use these statistics as measures of location and spread of a data distribution, being aware of their limitations. (ACMGM030)

Comparing data for a numerical variable across two or more groups:

- construct and use parallel box plots (including the use of the 'Q1 – 1.5 x IQR' and 'Q3 + 1.5 x IQR' criteria for identifying possible outliers) to compare groups in terms of location (median), spread (IQR and range) and outliers and to interpret and communicate the differences observed in the context of the data (ACMGM031)
- compare groups on a single numerical variable using medians, means, IQRs, ranges or standard deviations, as appropriate; interpret the differences observed in the context of the data; and report the findings in a systematic and concise manner (ACMGM032)
- implement the statistical investigation process to answer questions that involve comparing the data for a numerical variable across two or more groups; for example, are Year 11 students the fittest in the school? (ACMGM033)

Topic 2: Applications of trigonometry

Applications of trigonometry:

- review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle (ACMGM034)
- determine the area of a triangle given two sides and an included angle by using the rule $Area=\frac{1}{2}ab\sin C$, or given three sides by using Heron's rule, and solve related practical problems (ACMGM035)
- solve problems involving non-right-angled triangles using the sine rule (ambiguous case excluded) and the cosine rule (ACMGM036)
- solve practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of bearings in navigation. (ACMGM037)

Topic 3: Linear equations and their graphs

Linear equations:

- identify and solve linear equations (ACMGM038)
- develop a linear formula from a word description (ACMGM039)

Straight-line graphs and their applications:

- construct straight-line graphs both with and without the aid of technology (ACMGM040)
- determine the slope and intercepts of a straight-line graph from both its equation and its plot (ACMGM041)
- interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation (ACMGM042)
- construct and analyse a straight-line graph to model a given linear relationship; for example, modelling the cost of filling a fuel tank of a car against the number of litres of petrol required. (ACMGM043)

Simultaneous linear equations and their applications:

- solve a pair of simultaneous linear equations, using technology when appropriate (ACMGM044)
- solve practical problems that involve finding the point of intersection of two straight-line graphs; for example, determining the break-even point where cost and revenue are represented by linear equations. (ACMGM045)

Piece-wise linear graphs and step graphs:

- sketch piece-wise linear graphs and step graphs, using technology when appropriate (ACMGM046)
- interpret piece-wise linear and step graphs used to model practical situations; for example, the tax paid as income increases, the change in the level of water in a tank over time when water is drawn off at different intervals and for different periods of time, the charging scheme for sending parcels of different weights through the post. (ACMGM047)

Units 1 and 2 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics, in routine and non-routine problems in a variety of contexts • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models to solve routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics, in routine and non-routine problems • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems • selects and applies mathematical and statistical models to routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics, that apply to routine problems • selects and applies techniques in mathematics and statistics to solve routine problems • applies mathematical and statistical models to routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics • uses simple techniques in mathematics and statistics in routine problems • demonstrates familiarity with mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	• demonstrates limited familiarity with simple concepts of consumer arithmetic, algebra and matrices, linear equations, geometry and trigonometry, and statistics • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical or statistical models • uses digital technologies for arithmetic calculations and to display limited mathematical and statistical information

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments which are succinct and reasoned using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments which are clear and reasoned using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	• represents simple mathematical or statistical information in a structured context • communicates simple mathematical or statistical information • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

Unit 3

Unit Description

This unit has three topics: ‘Bivariate data analysis’, ‘Growth and decay in sequences’ and ‘Graphs and networks’.

‘Bivariate data analysis’ introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process.

‘Growth and decay in sequences’ employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population, or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4.

‘Graphs and networks’ introduces students to the language of graphs and the ways in which graphs, represented as a collection of points and interconnecting lines, can be used to model and analyse everyday situations such as a rail or social network.

Classroom access to technology to support the graphical and computational aspects of these topics is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in bivariate data analysis*,* growth and decay in sequences, and graphs and networks
- apply reasoning skills and solve practical problems in bivariate data analysis, growth and decay in sequences, and graphs and networks
- implement the statistical investigation process in contexts requiring the analysis of bivariate data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Content Descriptions

Topic 1: Bivariate data analysis

The statistical investigation process:

- review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results. (ACMGM048)

Identifying and describing associations between two categorical variables:

- construct two-way frequency tables and determine the associated row and column sums and percentages (ACMGM049)
- use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association (ACMGM050)
- describe an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data. (ACMGM051)

Identifying and describing associations between two numerical variables:

- construct a scatterplot to identify patterns in the data suggesting the presence of an association (ACMGM052)
- describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak) (ACMGM053)
- calculate and interpret the correlation coefficient (r) to quantify the strength of a linear association. (ACMGM054)

Fitting a linear model to numerical data:

- identify the response variable and the explanatory variable (ACMGM055)
- use a scatterplot to identify the nature of the relationship between variables (ACMGM056)
- model a linear relationship by fitting a least-squares line to the data (ACMGM057)
- use a residual plot to assess the appropriateness of fitting a linear model to the data (ACMGM058)
- interpret the intercept and slope of the fitted line (ACMGM059)
- use the coefficient of determination to assess the strength of a linear association in terms of the explained variation (ACMGM060)
- use the equation of a fitted line to make predictions (ACMGM061)
- distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation (ACMGM062)
- write up the results of the above analysis in a systematic and concise manner. (ACMGM063)

Association and causation:

- recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between them (ACMGM064)
- identify possible non-causal explanations for an association, including coincidence and confounding due to a common response to another variable, and communicate these explanations in a systematic and concise manner. (ACMGM065)

The data investigation process:

- implement the statistical investigation process to answer questions that involve identifying, analysing and describing associations between two categorical variables or between two numerical variables; for example, is there an association between attitude to capital punishment (agree with, no opinion, disagree with) and sex (male, female)? is there an association between height and foot length? (ACMGM066)

Topic 2: Growth and decay in sequences

The arithmetic sequence:

- use recursion to generate an arithmetic sequence (ACMGM067)
- display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations (ACMGM068)
- deduce a rule for the *n*th term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions (ACMGM069)
- use arithmetic sequences to model and analyse practical situations involving linear growth or decay; for example, analysing a simple interest loan or investment, calculating a taxi fare based on the flag fall and the charge per kilometre, or calculating the value of an office photocopier at the end of each year using the straight-line method or the unit cost method of depreciation. (ACMGM070)

The geometric sequence:

- use recursion to generate a geometric sequence (ACMGM071)
- display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations (ACMGM072)
- deduce a rule for the *n*th term of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions (ACMGM073)
- use geometric sequences to model and analyse (numerically, or graphically only) practical problems involving geometric growth and decay; for example, analysing a compound interest loan or investment, the growth of a bacterial population that doubles in size each hour, the decreasing height of the bounce of a ball at each bounce; or calculating the value of office furniture at the end of each year using the declining (reducing) balance method to depreciate. (ACMGM074)

Sequences generated by first-order linear recurrence relations:

- use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form (ACMGM075)
- recognise that a sequence generated by a first-order linear recurrence relation can have a long term increasing, decreasing or steady-state solution (ACMGM076)
- use first-order linear recurrence relations to model and analyse (numerically or graphically only) practical problems; for example, investigating the growth of a trout population in a lake recorded at the end of each year and where limited recreational fishing is permitted, or the amount owing on a reducing balance loan after each payment is made. (ACMGM077)

Topic 3: Graphs and networks

The definition of a graph and associated terminology:

- explain the meanings of the terms: graph, edge, vertex, loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph (digraph), arc, weighted graph, and network (ACMGM078)
- identify practical situations that can be represented by a network, and construct such networks; for example, trails

connecting camp sites in a National Park, a social network, a transport network with one-way streets, a food web, the results of a round-robin sporting competition (ACMGM079)

- construct an adjacency matrix from a given graph or digraph. (ACMGM080)

Planar graphs:

- explain the meaning of the terms: planar graph, and face (ACMGM081)
- apply Euler's formula, $v + f - e = 2$, to solve problems relating to planar graphs. (ACMGM082)

Paths and cycles:

- explain the meaning of the terms: walk, trail, path, closed walk, closed trail, cycle, connected graph, and bridge (ACMGM083)
- investigate and solve practical problems to determine the shortest path between two vertices in a weighted graph (by trial-and-error methods only) (ACMGM084)
- explain the meaning of the terms: Eulerian graph, Eulerian trail, semi-Eulerian graph, semi-Eulerian trail and the conditions for their existence, and use these concepts to investigate and solve practical problems; for example, the Königsberg Bridge problem, planning a garbage bin collection route (ACMGM085)
- explain the meaning of the terms: Hamiltonian graph and semi-Hamiltonian graph, and use these concepts to investigate and solve practical problems; for example, planning a sight-seeing tourist route around a city, the travelling-salesman problem (by trial-and-error methods only). (ACMGM086)

Unit 4

Unit Description

This unit has three topics: 'Time series analysis'; ' Loans, investments and annuities' and 'Networks and decision mathematics'.

'Time series analysis' continues students' study of statistics by introducing them to the concepts and techniques of time series analysis. The content is to be taught within the framework of the statistical investigation process.

'Loans and investments and annuities' aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments.

'Networks and decision mathematics' uses networks to model and aid decision making in practical situations.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Learning Outcomes

By the end of this unit, students:

- understand the concepts and techniques in time series analysis; loans, investments and annuities; and networks and decision mathematics
- apply reasoning skills and solve practical problems in time series analysis; loans, investments and annuities; and networks and decision mathematics
- implement the statistical investigation process in contexts requiring the analysis of time series data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Content Descriptions

Topic 1: Time series analysis

Describing and interpreting patterns in time series data:

- construct time series plots (ACMGM087)
- describe time series plots by identifying features such as trend (long term direction), seasonality (systematic, calendar-related movements), and irregular fluctuations (unsystematic, short term fluctuations), and recognise when there are outliers; for example, one-off unanticipated events. (ACMGM088)

Analysing time series data:

- smooth time series data by using a simple moving average, including the use of spreadsheets to implement this process (ACMGM089)
- calculate seasonal indices by using the average percentage method (ACMGM090)
- deseasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process (ACMGM091)
- fit a least-squares line to model long-term trends in time series data. (ACMGM092)

The data investigation process:

- implement the statistical investigation process to answer questions that involve the analysis of time series data. (ACMGM093)

Topic 2: Loans, investments and annuities

Compound interest loans and investments:

- use a recurrence relation to model a compound interest loan or investment, and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment (ACMGM094)
- calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly (ACMGM095)
- with the aid of a calculator or computer-based financial software, solve problems involving compound interest loans or investments; for example, determining the future value of a loan, the number of compounding periods for an investment to exceed a given value, the interest rate needed for an investment to exceed a given value. (ACMGM096)

Reducing balance loans (compound interest loans with periodic repayments):

- use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan (ACMGM097)
- with the aid of a financial calculator or computer-based financial software, solve problems involving reducing balance loans; for example, determining the monthly repayments required to pay off a housing loan. (ACMGM098)

Annuities and perpetuities (compound interest investments with periodic payments made from the investment):

- use a recurrence relation to model an annuity, and investigate (numerically or graphically) the effect of the amount invested, the interest rate, and the payment amount on the duration of the annuity (ACMGM099)
- with the aid of a financial calculator or computer-based financial software, solve problems involving annuities (including

perpetuities as a special case); for example, determining the amount to be invested in an annuity to provide a regular monthly income of a certain amount. (ACMGM100)

Topic 3: Networks and decision mathematics

Trees and minimum connector problems:

- explain the meaning of the terms tree and spanning tree identify practical examples (ACMGM101)
- identify a minimum spanning tree in a weighted connected graph either by inspection or by using Prim's algorithm (ACMGM102)
- use minimal spanning trees to solve minimal connector problems; for example, minimising the length of cable needed to provide power from a single power station to substations in several towns. (ACMGM103)

Project planning and scheduling using critical path analysis (CPA):

- construct a network to represent the durations and interdependencies of activities that must be completed during the project; for example, preparing a meal (ACMGM104)
- use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project (ACMGM105)
- use ESTs and LSTs to locate the critical path(s) for the project (ACMGM106)
- use the critical path to determine the minimum time for a project to be completed (ACMGM107)
- calculate float times for non-critical activities. (ACMGM108)

Flow networks

- solve small-scale network flow problems including the use of the 'maximum-flow minimum- cut' theorem; for example, determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank (the source) to a terminal (the sink). (ACMGM109)

Assignment problems

- use a bipartite graph and/or its tabular or matrix form to represent an assignment/ allocation problem; for example, assigning four swimmers to the four places in a medley relay team to maximise the team's chances of winning (ACMGM110)
- determine the optimum assignment(s), by inspection for small-scale problems, or by use of the Hungarian algorithm for larger problems. (ACMGM111)

Units 3 and 4 Achievement Standards

Concepts and Techniques

A	B	C	D	E
• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics in routine and non-routine problems in a variety of contexts • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems in a variety of contexts • develops, selects and applies mathematical and statistical models to routine and non-routine problems in a variety of contexts • uses digital technologies effectively to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems in a variety of contexts	• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics in routine and non-routine problems • selects and applies techniques in mathematics and statistics to solve routine and non-routine problems • selects and applies mathematical and statistical models to routine and non-routine problems • uses digital technologies appropriately to graph, display and organise mathematical and statistical information to solve a range of routine and non-routine problems	• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics that apply to routine problems • selects and applies techniques in mathematics and statistics to solve routine problems • applies mathematical and statistical models to routine problems • uses digital technologies to graph, display and organise mathematical and statistical information to solve routine problems	• demonstrates knowledge of concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics. • uses simple techniques in mathematics and statistics in routine problems • demonstrates familiarity with mathematical and statistical models • uses digital technologies to display some mathematical and statistical information in routine problems	• demonstrates limited familiarity with simple concepts of statistics, growth and decay in sequences, graphs and networks, and financial mathematics . • uses simple techniques in a structured context • demonstrates limited familiarity with mathematical or statistical models • uses digital technologies for arithmetic calculations and to display limited mathematical and statistical information

Reasoning and Communication

A	B	C	D	E
• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical and statistical judgments and arguments which are succinct and reasoned using appropriate language • interprets the solutions to routine and non-routine problems in a variety of contexts • explains the reasonableness of the results and solutions to routine and non-routine problems in a variety of contexts • identifies and explains the validity and limitations of models used when developing solutions to routine and non-routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine and non-routine problems • communicates mathematical and statistical judgments and arguments which are clear and reasoned using appropriate language • interprets the solutions to routine and non-routine problems • explains the reasonableness of the results and solutions to routine and non-routine problems • identifies and explains limitations of models used when developing solutions to routine problems	• represents mathematical and statistical information in numerical, graphical and symbolic form in routine problems • communicates mathematical and statistical arguments using appropriate language • interprets the solutions to routine problems • describes the reasonableness of the results and solutions to routine problems • identifies limitations of models used when developing solutions to routine problems	• represents simple mathematical and statistical information in numerical, graphical or symbolic form in routine problems • communicates simple mathematical and statistical information using appropriate language • describes solutions to routine problems • describes the appropriateness of the results of calculations • identifies limitations of simple models	• represents simple mathematical or statistical information in a structured context • communicates simple mathematical or statistical information • identifies solutions to routine problems • demonstrates limited familiarity with the appropriateness of the results of calculations • identifies simple models

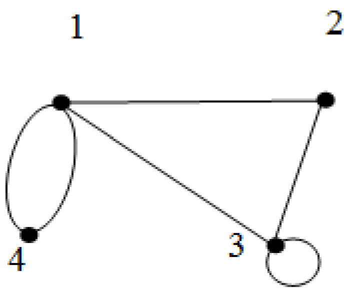
General Mathematics Glossary

Adjacency matrix

An adjacency matrix for a non-directed graph with n vertices is a $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j. In an adjacency matrix, a loop is counted as 1 edge.

Example:

Non-directed graph



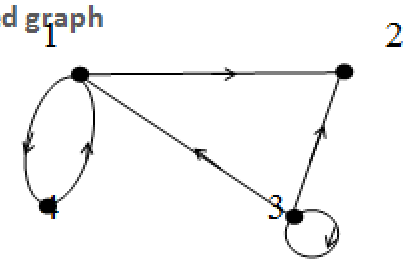
Adjacency matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

For a directed graph the entry in row i and column j is the number of directed edges (arcs) joining the vertex i and j in the direction i to j.

Example:

Directed graph



Adjacency matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Adjacent (graph)

see the definition of a graph on page 39

Algorithm

An algorithm is a precisely defined routine procedure that can be applied and systematically followed through to a conclusion. An example is Prim's algorithm for determining a minimum spanning tree in a network.

Arc see directed graph

Angle of depression

The angle a line makes below a horizontal plane.

Angle of elevation

The angle a line makes above a horizontal plane.

Annuity

An annuity is a compound interest investment from which payments are made on a regular basis for a fixed period of time. At the end of this time the investment has no residual value.

Area of a triangle

The general rule for determining the area of a triangle is: $\boldsymbol{area} = \frac{1}{2}\ \boldsymbol{base} \times \boldsymbol{height}$

Arithmetic sequence

An arithmetic sequence is a sequence of numbers such that the difference between any two successive members of the sequence is constant.

For example, the sequence

2, 5, 8, 11, 14, 17, …

is an arithmetic sequence with first term 2 and common difference 3.

By inspection of the sequence, the rule for the nth term t_n of this sequence is:

$t_n = 2 + (n - 1)3 = 3n - 1 \ n \geq 1$

If t_n is used to denote the nth term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2$, $t_{n+1} = t_n + 3 \ n \geq 1$

Association

A general term used to describe the relationship between two (or more) variables. The term association is often used interchangeably with the term correlation. The latter tends to be used when referring to the strength of a linear relationship between two numerical variables.

Average percentage method

In the average percentage method for calculating a seasonal index, the data for each 'season' are expressed as percentages of the average for the year. The percentages for the corresponding 'seasons' for different years are then averaged using a mean or median to arrive at a seasonal index.

Bearings (compass and true)

A bearing is the direction of a fixed point, or the path of an object, from the point of observation.

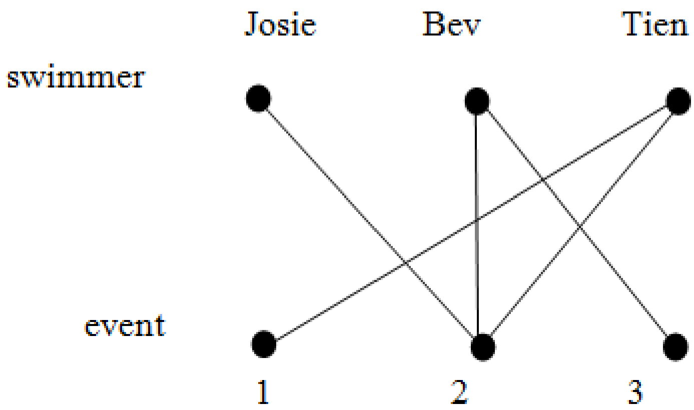
Compass bearings are specified as angles either side of north or south. For example a compass bearing of N50°E is found by facing north and moving through an angle of 50° to the East.

True (or three figure) bearings are measured in degrees from the north line. Three figures are used to specify the direction. Thus the direction of north is specified as 000°, east is specified as 090°, south is specified as 180° and north-west is specified as 315°.

Bipartite Graph

A bipartite graph is a graph whose set of vertices can be split into two distinct groups in such a way that each edge of the graph joins a vertex in the first group to a vertex in the second group.

Example:



Bridge see connected graph

Book value

The book value is the value of an asset recorded on a balance sheet. The book value is based on the original cost of the asset less depreciation.

For example, if the original cost of a printer is $500 and its value depreciates by $100 over the next year, then its book value at the end of the year is $400.

There are three commonly used methods for calculating yearly depreciation in the value of an asset, namely, reducing balancedepreciation, flat rate depreciation or unit cost depreciation.

Break-even point

The break-even point is the point at which revenue begins to exceed the cost of production.

Categorical data

Data associated with a categorical variable is called categorical data.

Categorical variable

A categorical variable is a variable whose values are categories.

Examples include blood group (A, B, AB or O) or house construction type (brick, concrete, timber, steel, other).

Categories may have numerical labels, eg. the numbers worn by player in a sporting team, but these labels have no numerical significance, they merely serve as labels.

Causation

A relationship between an explanatory and a response variable is said to be causal if the change in the explanatory variable actually causes a change in the response variable. Simply knowing that two variables are associated, no matter how strongly, is not sufficient evidence by itself to conclude that the two variables are causally related.

Possible explanations for an observed association between an explanatory and a response variable include:

the explanatory variable is actually causing a change in the response variable

there may be causation, but the change may also be caused by one or more uncontrolled variables whose effects cannot be disentangled from the effect of the response variable. This is known as confounding.

there is no causation, the association is explained by at least one other variable that is associated with both the explanatory and the response variable . This is known as a common response.

the response variable is actually causing a change in the explanatory variable

Closed path

See path

Closed trail

See trail

Closed walk

See walk

Coefficient of determination

In a linear model between two variables, the coefficient of determination (R^2) is the proportion of the total variation that can be explained by the linear relationship existing between the two variables, usually expressed as a percentage. For two variables only, the coefficient of determination is numerically equal to the square of the correlation coefficient (r^2).

Example

A study finds that the correlation between the heart weight and body weight of a sample of mice is $r = 0.765$. The coefficient of determination = $r^2 = 0.765^2 = 0.5852$ … or approximately 59%

From this information, it can be concluded that approximately 59% of the variation in heart weights of these mice can be explained by the variation in their body weights.

Note: The coefficient of determination has a more general and more important meaning in considering relationships between more than two variables, but this is not a school level topic.

Common response
See Causation

Complete graph

A complete graph is a simple graph in which every vertex is joined to every other vertex by an edge.

The complete graph with n vertices is denoted K_n .

Compound interest

The interest earned by investing a sum of money (the principal) is compound interest if each successive interest payment is added to the principal for the purpose of calculating the next interest payment.

For example, if the principal *P* earns compound interest at the rate of *i* % per period, then after *n* periods the total amount accrued is $P\left(1+\frac{i}{n}\right)^{n}$. When plotted on a graph, the total amount accrued is seen to grow exponentially.

Confounding

See Causation

Connected graph

A graph is connected if there is a path between each pair of vertices. A bridge is an edge in a connected graph that, if removed, leaves a graph disconnected.

Continuous data

Data associated with a continuous variable is called continuous data.

Continuous variable

A continuous variable is a numerical variable that can take any value that lies within an interval. In practice, the values taken are subject to accuracy of the measurement instrument used to obtain these values.

Examples include height, reaction time and systolic blood pressure.

Correlation

Correlation is a measure of the strength of the linear relationship between two variables. See also association.

Correlation coefficient (r)

The correlation coefficient (*r*) is a measure of the strength of the liner relationship between a pair of variables. The formula for calculating *r* is given below.

For variables *x* and *y*, and computed for *n* cases, the formula for *r* is:

$$r=\frac{1}{n-1}\ \sum\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right)$$

Cosine rule

For a triangle of side lengths *a*, *b* and *c* and angles *A*, *B* and *C*, the cosine rule states that

$c^2 = a^2 + b^2 - 2ab \cos C$

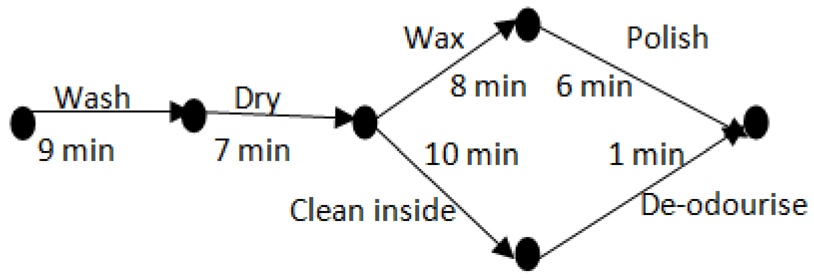
CPI

The Consumer Price Index (CPI) is a measure of changes, over time, in retail prices of a constant basket of goods and services representative of consumption expenditure by resident households in Australian metropolitan areas.

Critical path analysis (CPA)

A project often involves many related activities some of which cannot be started until one or more earlier tasks have been completed. One way of scheduling such activities that takes this into account is to construct a network diagram.

The network diagram below can be used to schedule the activities of two or more individuals involved in cleaning and polishing a car. The completion times for each activity are also shown.



Critical path analysis is a method for determining the longest path (the critical path) in such a network and hence the minimum time in which the project can be completed. There may be more that one critical path in the network. In this project the critical path is ‘Wash-Dry-Wax-Polish’ with a total completion time of 30 minutes.

The earliest starting time (EST) of an activity ‘Polish’ is 24 minutes because activities ‘Wash’, ‘Dry’ and ‘Wax’ must be completed first. The process of systematically determining earliest starting times is called forward scanning.

The shortest time that the project can be completed is 30 minutes. Thus, the latest starting time (LST) for the activity ‘De-odourise’ is 29 minutes. The process of systematically determining latest starting times is called backward scanning.

Float or slack

Is the amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks. For example, the activity ‘De-odourise’ is said to have a float of 3 minutes because its earliest EST (26 minutes) is three minutes before its LST (29 minutes). As a result this activity can be started at any time between 26 and 29 minutes after the project started. All activities on a critical path have zero floats.

Cut (in a flow network)

In a flow network, a cut is a partition of the vertices of a graph into two separate groups with the source in one group and the sink in the other.

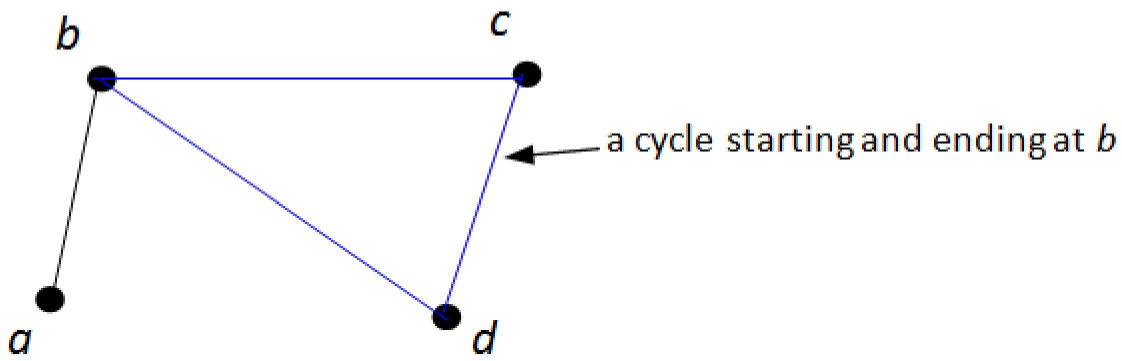
The capacity of the cut is the sum of the capacities of the cut edges directed from source to sink. Cut edges directed from sink to source are ignored.

Example:

$\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{4}$, $\frac{6}{4}$, $\frac{3}{8}$, $\frac{2}{2}$, $\frac{3}{2}$

Cycle

A cycle is a closed walk begins and starts at the same vertex and in which has no repeated edges or vertices except the first.. If *a*, *b*, *c* and *d* are the vertices of a graph, the closed walk *bcdb* that starts and ends at vertex *b* (shown in blue) an example of a cycle.



Degree of a vertex (graph)

In a graph, the degreeof a vertex is the number of edges incident with the vertex, with loops counted twice. It is denoted deg *v*.

In the graph below, deg *a* = 4, deg *b* = 2, deg *c* = 4 and deg *d*=2.

Digraph

See directed graph

Directed graph

A directed graph is a diagram comprising points, called vertices, joined by directed lines called arcs. The directed graphs are commonly called digraphs.

Discrete data

Discrete data is data associated with a discrete variable. Discrete data is sometimes called count data.

Discrete variable

A discrete variable is a numerical variable that can take only integer values.

Examples include the number of people in a car, the number of decayed teeth in 18 year-old males, etc.

Earliest starting time (EST)

See Critical Path Analysis

Edge

See graph

Effective annual rate of interest

The effective annual rate of interest $i_{\text{effective}}$ is used to compare the interest paid on loans (or investments) with the same nominal annual interest rate *i* but with different compounding periods (daily, monthly, quarterly, annually, other)

If the number of compounding periods per annum is *n*, then $i_{effective} = \left(1 + \frac{i}{n}\right)^n - 1$

For example if the quoted annual interest rate for a loan is 9%, but interest is charged monthly, then the effective annual interest rate charged is $i_{effective} = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 0.9416$, or around 9.4%.

Diminishing value depreciation see Reducing balance depreciation

Elements (Entries) of a matrix

The symbol a_{ij} represents the (*i,j*) element occurring in the i^{th} row and the j^{th} column.

For example a general 3 × 2 matrix is:

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ where a_{32} is the element in the third row and the second column

Euler's formula

For a connected planar graph, Euler's rule states that

$v + f - e = 2$

where *v* is the number vertices, *e* the number of edges and *f* is the number of faces.

Eulerian

A connected graph is Eularian if it has a closed trail (starts and ends at the same vertex), that is, includes every edge and once only; such a trail is called an Eulerian trail. An Eularian trial may include repeated vertices. A connected graph is semi-Eularian if there is an open trail that includes every once only.

Explanatory variable

When investigating relationships in bivariate data, the explanatory variable is the variable used to explain or predict a difference in the response variable.

For example, when investigating the relationship between the temperature of a loaf of bread and the time it has spent in a hot oven, *temperature* is the response variable and *time* is the explanatory variable.

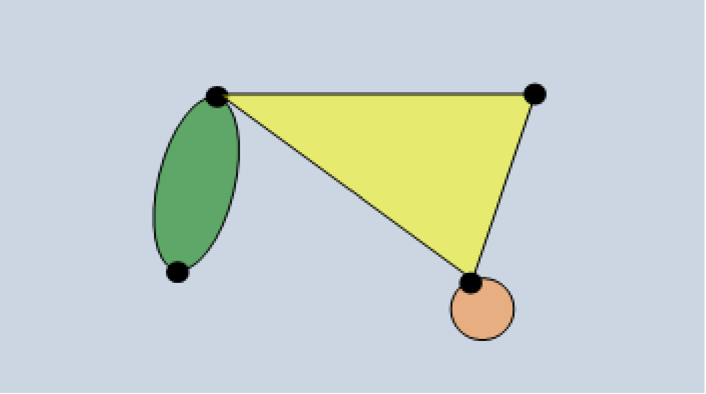
Extrapolation

In the context of fitting a linear relationship between two variables, extrapolation occurs when the fitted model is used to make predictions using values of the explanatory variable that are outside the range of the original data. Extrapolation is a dangerous process as it can sometimes lead to quite erroneous predictions.

See also interpolation.

Face

The faces of a planar graph are the regions bounded by the edges including the outer infinitely large region. The planar graph shown has four faces.



First-order linear recurrence relation

A first-order linear recurrence relation is defined by the rule: $t_0 = a$,$t_{n+1} = bt_n + c$ for $n \geq 1$

For example, the rule: $t_0 = 10$, $t_n = 5t_{n-1} + 1$ for $n \geq 1$ is a first-order recurrence relation.

The sequence generated by this rule is: 10, 51, 256, … as shown below.

$t_1 = 10$, $t_2 = 5t_1 + 1 = 5 \times 10 + 1 = 51$, $t_3 = 5t_2 + 1 = 5 \times 51 + 1 = 256$, …

Five-number summary

A five-number summary is a method of summarising a set of data using the minimum value, the lower or first-quartile (Q_1), the median, the upper or third-quartile (Q_3) and the maximum value. Forms the basis for a boxplot.

Flat rate depreciation

In flat rate or straight-line depreciation the value of an asset is depreciated by a fixed amount each year. Usually this amount is specified as a fixed percentage of the original cost.

Float time

See Critical Path Analysis

Flow network

A flow network is a directed graph where each edge has a capacity (e.g. 100 cars per hour, 800 litres per minute, etc) and each edge receives a flow. The amount of flow on an edge cannot exceed the capacity of the edge. A flow must satisfy the restriction that the amount of flow into a node equals the amount of flow out of it, except when it is a source, which has more outgoing flow, or a sink, which has more incoming flow. A flow network can be used to model traffic in a road system, fluids in pipes, currents in an electrical circuit, or any situation in which something travels through a network of nodes.

Food web

A food web (or food chain) depicts feeding connections (who eats whom) in an ecological community.

Geometric growth or decay (sequence)

A sequence displays geometric growth or decay when each term is some constant multiple (greater or less than one) of the preceding term. A multiple greater than one corresponds to growth. A multiple less than one corresponds to decay.

For example, the sequence:

1, 2, 4, … displays geometric growth because each term is double the previous term.

100, 10, 0.1, … displays geometric decay because each term is one tenth of the previous term.

Geometric growth is an example of exponential growth in discrete situations.

Geometric sequence

A geometric sequence, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed non-zero number called the common ratio. For example, the sequence

2, 6, 18, ...

is a geometric sequence with first term 2 and common ratio 3.

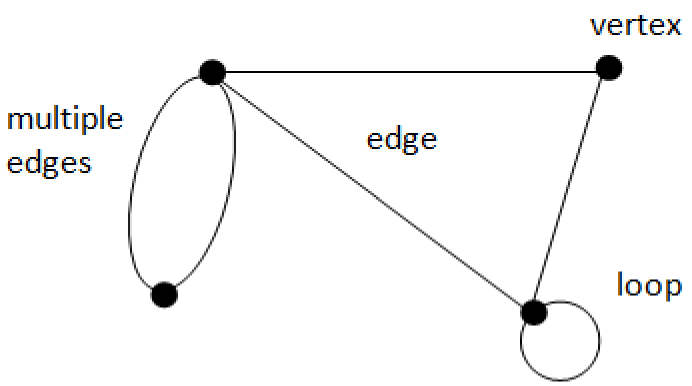
By inspection of the sequence, the rule for the *n*th term of this sequence is:

$t_n = 2 \times 3^{n-1} n \geq 1$

If t_n is used to denote the *n*th term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2$, $t_{n+1} = 3t_n$ $n \geq 1$

Graph

A graph is a diagram that consists of a set of points, called vertices that are joined by a set of lines called edges. Each edge joins two vertices. A loop is an edge in a graph that joins a vertex in a graph to itself. Two vertices are adjacent if they a joined by an edge. Two or more edges connect the same vertices are called multiple edges.



GST

The GST (Goods and Services Tax) is a broad sales tax of 10% on most goods and services transactions in Australia.

Hamiltonian

A Hamiltonian cycle is a cycle that includes each vertex in a graph (except the first), once only.
A Hamilton path is path that includes every vertex in a graph once only. A Hamilton path that begins and ends at the same vertex is a Hamiltonian cycle.

Heron's rule

Heron's rule is a rule for determining the area of a triangle given the lengths of its sides.

The area *A* of a triangle of side lengths *a*, *b* and *c* is given by

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $= \frac{1}{2}\left(a+b+c\right)$.

Hungarian algorithm

The Hungarian algorithm is used to solve assignment (allocation) problems.

Identity matrix

A multiplicative identity matrix is a square matrix in which all of the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter *I*.

For example,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ are both identity matrices.

.

There is an identity matrix for each size (or order) of a square matrix. When clarity is needed, the order is written with a subscript: I_n

Interpolation

In the context of fitting a linear relationship between two variables, interpolation occurs when the fitted model is used to make predictions using values of the explanatory variable that lie within the range of the original data.

See also extrapolation.

Inverse of a 2 × 2 matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Is

$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ provided $ad - bc \neq 0$

Inverse of a square matrix

The inverse of a square matrix*A* is written as A^{-1} and has the property that

$AA^{-1} = A^{-1}A = I$

Not all square matrices have an inverse. A matrix that has an inverse is said to be invertible.

Irregular variation or noise (time series)

Irregular variation or noise is erratic and short-term variation in a time series that is the product of chance occurrences.

Königsberg Bridge problem

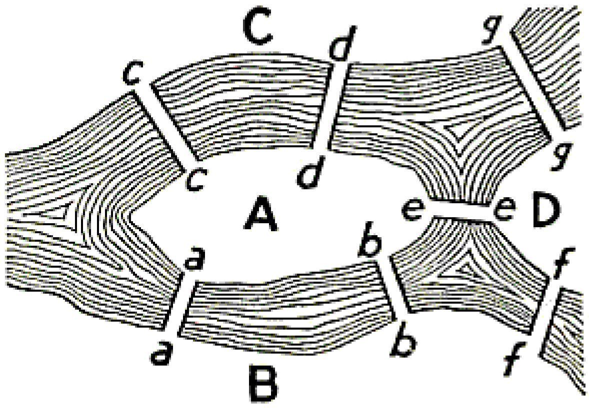


FIGURE 98. *Geographic Map: The Königsberg Bridges.*

The Königsberg bridge problem asks: Can the seven bridges of the city of Königsberg all be traversed in a single trip that starts and finishes at the same place?

Latest starting time (LST)

See Critical Path Analysis

Leading diagonal

The leading diagonal of a square matrix is the diagonal that runs from the top left corner to the bottom right corner of the matrix.

Least-squares line

In fitting a straight-line $y = a + bx$ to the relationship between a response variable y and an explanatory variable x, the least-squares line is the line for which the sum of the squared residuals is the smallest.

The formula for calculating the slope (b) and the intercept (a) of the least squares line is given below.

For variables x and y computed for n cases, the slope (b) and intercept (a) of the least-squares line are given by:

$b = \frac{\sum(x_i - \overline{x})(y_i - \overline{y})}{\sum(x_i - \overline{x})^2}$ *or* $b = r\,\frac{s_y}{s_x}$ *and* $a = \overline{y} - b\overline{x}$

Length (of a walk)

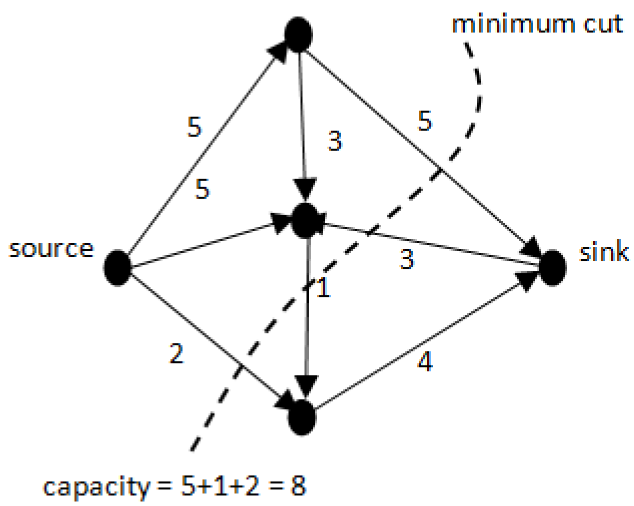
The length of a walk is the number of edges it includes.

Minimum cut-maximum flow theorem

The maximum flow–minimum cut theorem states that in a flow network, the maximum flow from the source to the sink is equal to the capacity of the minimum cut.

In everyday language, the minimum cut involves identifies the ‘bottle-neck’ in the system.

Example:



Linear equation

A linear equation in one variable x is an equation of the form $ax + b = 0$, e.g. $3x + 1 = 0$

A linear equation in two variables x and y is an equation of the form $ax + by + c = 0$,

e.g. $2x – 3y + 5 = 0$

Linear graph

A linear graph is a graph of a linear equation with two variables. If the linear equation is written in the form $y = a + bx$, then a represents the y-intercept and b represents the slope (or gradient) of the linear graph.

Linear growth or decay (sequence)

A sequence displays linear growth or decay when the difference between successive terms is constant. A positive constant difference corresponds to linear growth while a negative constant difference corresponds to decay.

Examples:

The sequence, 1, 4, 7, … displays linear growth because the difference between successive terms is 3.

The sequence, 100, 90, 80, … displays linear decay because the difference between successive terms is –10. By definition, arithmetic sequences display linear growth or decay.

Location

The notion of central or 'typical value' in a sample distribution.

See also mean, median and mode.

Matrix (matrices)

A matrix is a rectangular array of elements or entities displayed in rows and columns.

For example,

$A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ are both matrices with six elements.

Matrix *A* is said to be a 3×2 matrix (three rows and two columns) while *B* is said to be a 2 × 3 matrix (two rows and three columns).

A square matrix has the same number of rows and columns.

A column matrix (or vector) has only one column.

A row matrix (or vector) has only one row.

Matrix multiplication

Matrix multiplication is the process of multiplying a matrix by another matrix.

For example, forming the product

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 25 \\ 11 & 45 \end{bmatrix}$$

The multiplication is defined by 1 × 2 + 8 × 0 + 0 × 4 = 2

1 × 1 + 8 × 3 + 0 × 4 = 25

2 × 2 + 5 × 0 + 7 × 1 = 11

2 × 1 + 5 × 3 + 7 × 4 = 45

This is an example of the process of matrix multiplication.

The product *AB* of two matrices *A* and *B* of size $m \times n$ and $p \times q$ respectively is defined if $n = p$.

If $n = p$ the resulting matrix has size $m \times q$.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & ba_{23} \end{bmatrix}$ then

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{12}b_{12} + a_{22}b_{22} & a_{12}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$$

Order (of a matrix)

See size (of a matrix)

Scalar multiplication (matrices)

Scalar multiplication is the process of multiplying a matrix by a scalar (number).

For example, forming the product

$$10\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 0 & 30 \\ 10 & 40 \end{bmatrix}$$

is an example of the process of scalar multiplication.

In general for the matrix *A* with elements a_{ij} the elements of *kA* are ka_{ij}.

Mean

The arithmetic mean of a list of numbers is the sum of the data values divided by the number of values in the list.

In everyday language, the arithmetic mean is commonly called the average.

For example, for the following list of five numbers 2, 3, 3, 6, 8 the mean equals

$$\frac{2+3+3+6+8}{5} = \frac{22}{5} = 4.4$$

In more general language, the mean of n observations $x_1, x_2, \ldots, x_n$ is $\overline{x} = \frac{\sum x_i}{n}$

Median

The median is the value in a set of ordered set of data values that divides the data into two parts of equal size. When there are an odd number of data values, the median is the middle value. When there is an even number of data values, the median is the average of the two central values.

Minimum spanning tree

For a given connected weighted graph, the minimum spanning tree is the spanning tree of minimum length.

Multiple edges

See graph

Mode

The mode is the most frequently occurring value is a data set.

Moving average

In a time series, a simple moving average is a method used to smooth the time series whereby each observation is replaced by a simple average of the observation and its near neighbours. This process reduces the effect of non-typical data and makes the overall trend easier to see.

Note: There are times when it is preferable to use a weighted average rather simple average, but this is not required in the current curriculum.

Network

The word network is frequently used in everyday life, e.g. television network, rail network, etc. Weighted graphs or digraphs can often be used to model such networks.

Open path

See path

Open trail

See trail

Open walk

See walk

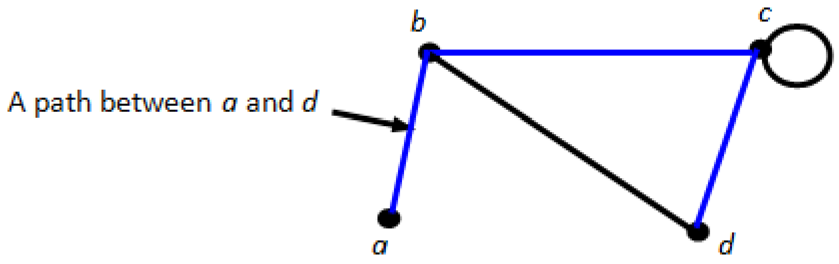
Outlier

An outlier in a set of data is an observation that appears to be inconsistent with the remainder of that set of data. An outlier is a surprising observation.

Path (in a graph)

A path in a graph is a walk in which all of the edges and all the vertices are different.. A path that starts and finishes at different vertices is said to be open, while a path that stats and finishes at the same vertex is said to be closed. A cycle is a closed path.

If *a* and *d* are the vertices of a graph, a walk from *a* to *d* along the edges coloured blue is a path. Depending on the graph, there may be multiple paths between the same two vertices, as is the case here.



Perpetuity

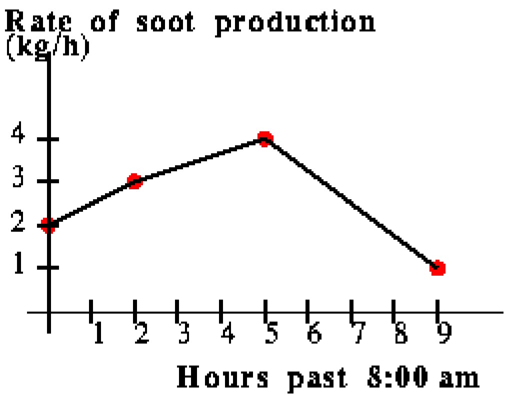
A perpetuity is a compound interest investment from which payments are made on a regular basis in perpetuity (forever). This is possible because the payments made at the end of each period exactly equal the interest earned during that period.

Piecewise-linear graph

A graph consisting of one or more none overlapping line segments.

Sometimes called a line segment graph.

Example:



Planar graph

A planar graph is a graph that can be drawn in the plane. A planar graph can always be drawn so that no two edges cross.

Price to earnings ratio (of a share)

The price to earnings ratio of a share (P/E ratio) is defined as :

$$P/Eratio = \frac{Market\ price\ per\ share}{Annual\ earnings\ per\ share}$$

Prim's algorithm

An algorithm for determining a minimum spanning tree in a connected weighted graph.

Recurrence relation

A recurrence relation is an equation that recursively defines a sequence; that is, once one or more initial terms are given, each further term of the sequence is defined as a function of the preceding terms.

Recursion

See recurrence relation

Reducing balance depreciation

In reducing balance depreciation the value of an asset is depreciated by a fixed percentage of its value each year.

Reducing balance depreciation is sometimes called diminishing value depreciation.

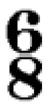
Reducing balance loan

A reducing balance loan is a compound interest loan where the loan is repaid by making regular payments and the interest paid is calculated on the amount still owing (the reducing balance of the loan) after each payment is made.

Residual plot

A residual plot is a scatterplot with the residual values shown on the vertical axis and the explanatory variable shown on the horizontal axis. Residual plots are useful in assessing the fit of the statistical model (e.g., by a least-squares line).

When the least-squares line captures the overall relationship between the response variable y and the explanatory variable x, the residual plot will have no clear pattern (be random) see opposite. This is what is hoped for.



scatterplot with least squares line

residual plot

If the least-squares line fails to capture the overall relationship between a response variable and an explanatory variable, a residual plot will reveal a pattern in the residuals. A residual plot will also reveal any outliers that may call into question the use of a least-squares line to describe the relationship. Interpreting patterns in residual plots is a skilled art and is not required in this curriculum.

Residual values

The difference between the observed value and the value predicted by a statistical model (e.g., by a least-squares line)

Response variable

See Explanatory variable

Round-robin sporting competition

A single round robin sporting competition is a competition in which each competitor plays each other competitor once only.

Scale factor

A scale factor is a number that scales, or multiplies, some quantity. In the equation $y = kx$, k is the scale factor for x.

If two or more figures are similar, their sizes can be compared. The scale factor is the ratio of the length of one side on one figure to the length of the corresponding side on the other figure. It is a measure of magnification, the change of size.

Scatterplot

A two-dimensional data plot using Cartesian co-ordinates to display the values of two variables in a bivariate data set.

For example the scatterplot below displays the CO_2 emissions in tonnes per person (*co2*) plotted against Gross Domestic Product per person in $US (*gdp*) for a sample of 24 countries in 2004. In constructing this scatterplot, gdp has been used as the explanatory variable.

Seasonal adjustment (adjusting for seasonality)

A term used to describe a time series from which periodic variations due to seasonal effects have been removed.

See also seasonal index.

Seasonal index

The seasonal index can be used to remove seasonality from data. An index value is attached to each period of the time series within a year. For the seasons of the year (Summer, Autumn, Winter, Spring) there are four separate seasonal indices; for months, there are 12 separate seasonal indices, one for each month, and so on. There are several methods for determining seasonal indices.

Seasonal variation

A regular rise and fall in the time series that recurs each year.

Seasonal variation is measured in terms of a seasonal index.

Smoothing (time series) see moving average

Semi-Eularian graph

See Eularian graph

Sequence

A sequence is an ordered list of numbers (or objects).

For example 1, 3, 5, 7 is a sequence of numbers that differs from the sequence 3, 1, 7, 5 as order matters.

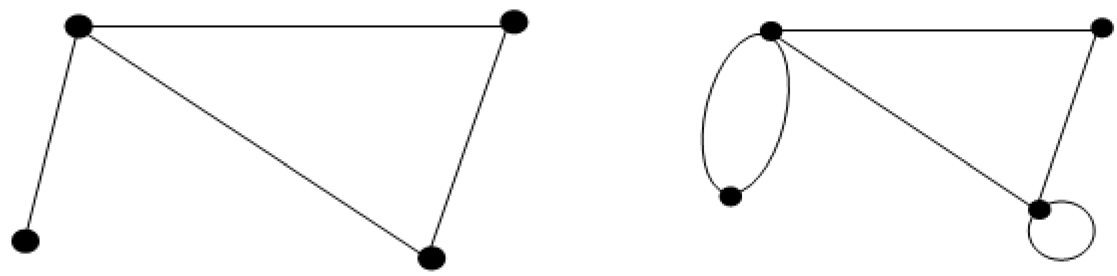
A sequence maybe finite, for example, 1, 3, 5, 7 (the sequence of the first four odd numbers), or infinite, for example, 1, 3, 5, …
(the sequence of all odd numbers).

Similar figures

Two geometric figures are similar if they are of the same shape but not necessarily of the same size.

Simple graph

A simple graph has no loops or multiple edges.



simple graph   non-simple graph

Simple interest

Simple interest is the interest accumulated when the interest payment in each period is a fixed fraction of the principal. For example, if the principle *P* earns simple interest at the rate of *i* % per period, then after *n* periods the accumulated simple interest is $nP\,\frac{i}{100}$

When plotted on a graph, the total amount accrued is seen to grow linearly.

Sine rule

For a triangle of side lengths *a*, *b* and *c* and angles *A*, *B* and *C*, the sine rule states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Singular matrix

A matrix is singular if det *A* = 0. A singular matrix does not have a multiplicative inverse.

Size (of a matrix)

Two matrices are said to have the same size (or order) if they have the same number of rows and columns. A matrix with *m* rows and *n* columns is said to be a *m* × *n* matrix.

For example, the matrices

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \text{ and } \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

have the same size. They are both 2 × 3 matrices.

Slope (gradient)

The slope or gradient of a line describes its steepness, incline, or grade.

Slope is normally described by the ratio of the "rise" divided by the "run" between two points on a line.

See also linear graph.

Spanning tree

A spanning tree is a subgraph of a connected graph that connects all vertices and is also a tree.

Standard deviation

The standard deviation is a measure of the variability or spread of a data set. It gives an indication of the degree to which the individual data values are spread around their mean.

The standard deviation of n observations $x_1, x_2, \ldots, x_n$ is

$$s = \sqrt{\frac{\sum\left(x_i - \overline{x}\right)^2}{n-1}}$$

Statistical investigation process

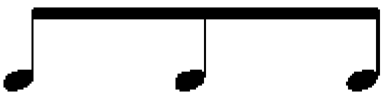
The statistical investigation process is a cyclical process that begins with the need to solve a real world problem and aims to reflect the way statisticians work. One description of the statistical investigation process in terms of four steps is as follows.

Step 1. Clarify the problem and formulate one or more questions that can be answered with data.

Step 2. Design and implement a plan to collect or obtain appropriate data.

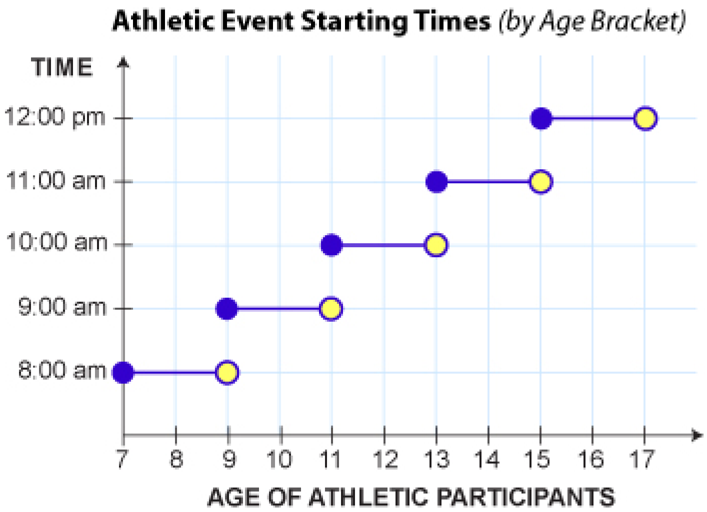
Step 3. Select and apply appropriate graphical or numerical techniques to analyse the data.

Step 4. Interpret the results of this analysis and relate the interpretation to the original question; communicate findings in a systematic and concise manner.



Step graph

A graph consisting of one or more non-overlapping horizontal line segments that follow a step-like pattern.



Matrices

Addition of matrices

If A and B are matrices of the same size (order) and the elements of A are a_{ij} and the elements of B are b_{ij} then the elements of $A + B$ are $a_{ij} + b_{ij}$

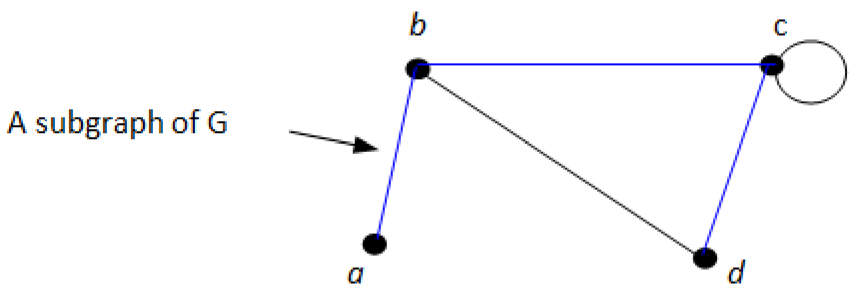
For example if $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 6 \end{bmatrix}$ Then $A + B = \begin{bmatrix} 7 & 2 \\ 2 & 4 \\ 2 & 10 \end{bmatrix}$

Straight-line depreciation

See: flat rate depreciation

Subgraph

When the vertices and edges of a graph A (shown in blue) are the vertices and edges of the graph G, graph A is said to be a subgraph of graph G.



Time series

Values of a variable recorded, usually at regular intervals, over a period of time. The observed movement and fluctuations of many such series comprise long-term trend, seasonal variation, and irregular variation or noise.

Time series plot

The graph of a time series with time plotted on the horizontal axis.

Trail

A trail is a walk in which no edge is repeated.

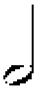
The travelling salesman problem

The travelling salesman problem can be described as follows: Given a list of cities and the distance between each city, find the shortest possible route that visits each city exactly once.

While in simple cases this problem can be solved by systematic identification and testing of possible solutions, no there is no known efficient method for solving this problem.

Tree

A tree is a connected graph with no cycles.



Trend (time series)

Trend is the term used to describe the general direction of a time series (increasing/ decreasing) over a long period of time.

Triangulation

The process of determining the location of a point by measuring angles to it from known points at either end of a fixed baseline, rather than measuring distances to the point directly. The point can then be fixed as the third point of a triangle with one known side and two known angles.

Two-way frequency table

A two-way frequency table is commonly used for displaying the two-way frequency distribution that arises when a group of individuals or objects are categorised according to two criteria.

For example, the two-way table below displays the frequency distribution that arises when 27 children are categorised according to *hair type* (straight or curly) and *hair colour* (red, brown, blonde, black).

Hair colour	Hair type		Total
	Straight	Curly	
red	1	1	2
brown	8	4	12
blonde	1	3	4
black	7	2	9
Total	17	10	27

The row and column totals represent the total number of observations in each row and column and are sometimes called row sums or column sums.

If the table is 'percentaged' using row sums the resulting percentages are called row percentages. If the table is 'percentaged' using row sums the resulting percentages are called column percentages.

Unit cost depreciation

In unit cost depreciation, the value of an asset is depreciated by an amount related to the number of units produced by the asset during the year.

Geometry and trigonometry

Vertex

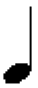
See graph

Walk (in a graph)

A walk in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. A walk that starts and finishes at different vertices is said to be an openwalk. A walk that starts and finishes at the same vertex is said to be closed walk.

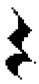
If *a*, *b*, *c* and *d* are the vertices of a graph with edges *ab*, *bc*, *cc*, *cd* and *bd*, then the sequence of edges (*ab*, *bc*, *cc*, *cd*) constitute a walk. The route followed on this walk is shown in blue on the graph below.

This walk is denoted by the sequence of vertices *abccd*. The walk is open because it begins and finishes at different vertices.



A walk can include repeated vertices (as is the case above) or repeated edges.

A example of a closed walk with both repeated edges and hence vertices is defined by the sequence of edges (*ab*, *bd*, *db*, *ba*) and is denoted by the sequence of vertices *abdba*. The route followed is shown in red in the graph below.



Depending on the graph, there may be multiple walks between the same two vertices, as is the case here.

Weighted graph

A weighted graph is a graph in which each edge is labelled with a number used to represent some quantity associated with the edge. For example, if the vertices represent towns, the weights on the edges may represent the distances in kilometres between the towns.

Zero matrix

A zero matrix is a matrix if all of its entries are zero. For example:

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ And $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ are zero matrices.

Statistics

Glossary

Abstract

Abstract scenario: a scenario for which there is no concrete referent provided.

Account

Account for: provide reasons for (something).

Give an account of: report or describe an event or experience.

Taking into account: considering other information or aspects.

Analyse

Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences.

Apply

Use, utilise or employ in a particular situation.

Assess

Determine the value, significance or extent of (something).

Coherent

Orderly, logical, and internally consistent relation of parts.

Communicates

Conveys knowledge and/or understandings to others.

Compare

Estimate, measure or note how things are similar or dissimilar.

Complex

Consisting of multiple interconnected parts or factors.

Considered

Formed after careful thought.

Critically analyse

Examine the component parts of an issue or information, for example the premise of an argument and its plausibility, illogical reasoning or faulty conclusions

Critically evaluate

Evaluation of an issue or information that includes considering important factors and available evidence in making critical judgement that can be justified.

Deduce

Arrive at a conclusion by reasoning.

Demonstrate
Give a practical exhibition as an explanation.

Describe
Give an account of characteristics or features.

Design
Plan and evaluate the construction of a product or process.

Develop
In history: to construct, elaborate or expand.

In English: begin to build an opinion or idea.

Discuss
Talk or write about a topic, taking into account different issues and ideas.

Distinguish
Recognise point/s of difference.

Evaluate
Provide a detailed examination and substantiated judgement concerning the merit, significance or value of something.

In mathematics: calculate the value of a function at a particular value of its independent variables.

Explain
Provide additional information that demonstrates understanding of reasoning and/or application.

Familiar
Previously encountered in prior learning activities.

Identify
Establish or indicate who or what someone or something is.

Integrate
Combine elements.

Investigate
Plan, collect and interpret data/information and draw conclusions about.

Justify
Show how an argument or conclusion is right or reasonable.

Locate
Identify where something is found.

Manipulate
Adapt or change.

Non-routine

Non-routine problems: Problems solved using procedures not previously encountered in prior learning activities.

Reasonableness

Reasonableness of conclusions or judgements: the extent to which a conclusion or judgement is sound and makes sense

Reasoned

Reasoned argument/conclusion: one that is sound, well-grounded, considered and thought out.

Recognise

Be aware of or acknowledge.

Relate

Tell or report about happenings, events or circumstances.

Represent

Use words, images, symbols or signs to convey meaning.

Reproduce

Copy or make close imitation.

Responding

In English: When students listen to, read or view texts they interact with those texts to make meaning. Responding involves students identifying, selecting, describing, comprehending, imagining, interpreting, analysing and evaluating.

Routine problems

Routine problems: Problems solved using procedures encountered in prior learning activities.

Select

Choose in preference to another or others.

Sequence

Arrange in order.

Solve

Work out a correct solution to a problem.

Structured

Arranged in a given organised sequence.

In Mathematics: When students provide a structured solution, the solution follows an organised sequence provided by a third party.

Substantiate

Establish proof using evidence.

Succinct

Written briefly and clearly expressed.

Sustained

Consistency maintained throughout.

Synthesise

Combine elements (information/ideas/components) into a coherent whole.

Understand

Perceive what is meant, grasp an idea, and to be thoroughly familiar with.

Unfamiliar

Not previously encountered in prior learning activities.