

# Fluency

## Portfolio summary

In F–2, students become fluent as they develop skills in choosing appropriate procedures; and recalling factual knowledge and concepts readily.

In Years 3–6, students become fluent as they develop skills in choosing appropriate procedures; carrying out procedures flexibly and accurately; and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, and when they recall definitions and regularly use facts.

In Years 7–8, students develop skills in choosing appropriate procedures; carrying out procedures flexibly, accurately, efficiently and appropriately; and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, and when they recall definitions and regularly use facts.

In Years 9–10, students develop skills in choosing appropriate procedures; carrying out procedures flexibly, accurately, efficiently and appropriately; and recalling factual knowledge and concepts readily. Students are fluent when they calculate answers efficiently, when they recognise robust ways of answering questions, when they choose appropriate methods and approximations, when they recall definitions and regularly use facts, and when they can manipulate expressions and equations to find solutions.

## Number and algebra: Who are we?

### Sample summary

The learning intention of the task was to identify given numbers which occur before and after a given number on a hundreds number chart.

### Proficiencies

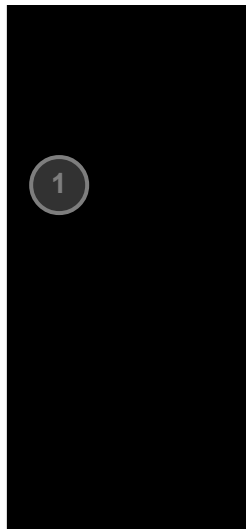
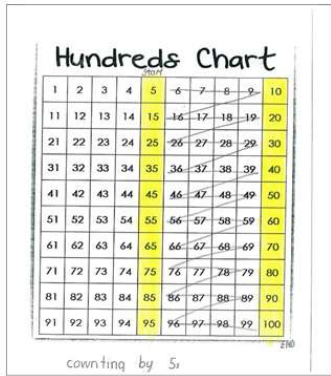
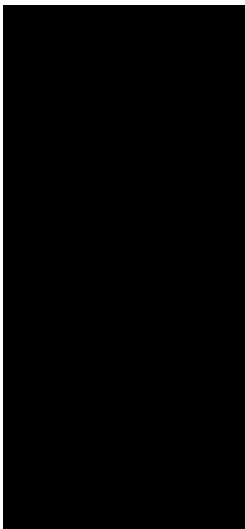
#### Understanding

#### Fluency

#### Reasoning

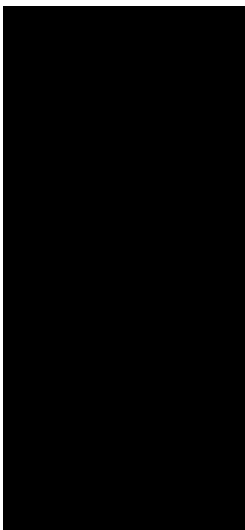
At this year level understanding includes connecting names, numerals and quantities, and partitioning numbers in various ways.

#### Hundreds chart



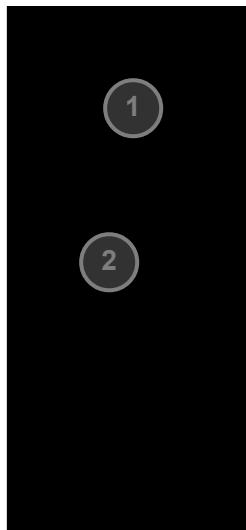
### Annotations

- 1 **Fluency**  
Carries out skip counting by fives from zero accurately



I have been counting by 5s and how you count by 5s is you start at 5 then you go to the number ten then after that you go to fifteen I will show you how to count by 5s.

5 10 15 20 25 30 35 40 45  
50 55 60 65 70 75 80 85 90  
95 100.



### Annotations

- 1 **Understanding**  
Identifies and represents appropriately on a hundreds chart the sequence of numbers (multiples of five) obtained
- 2 **Reasoning**  
Explains the strategy used to skip count by fives from zero and demonstrates the results of using the strategy by listing the appropriate sequence of numbers to 100

## Number and algebra: Fractions and decimals and percentages

### Sample summary

Students were given the following problem to solve:

'A school is enclosed by a fence that has sides of equal length. 60% of the whole fence has been painted black. How many different ways can you draw what the fence might look like? Look at each of your drawings and represent the painted section as a fraction. Represent it as a decimal.'

### Proficiencies

#### Understanding

# Fluency

## Problem-Solving

### Reasoning

At this year level understanding includes describing properties of different sets of numbers, using fractions and decimals to describe probabilities, representing fractions and decimals in various ways and describing connections between them, and making reasonable estimations.

#### Worksheet

The student's work is on graph paper and includes the following:

- Annotation 1:** A calculation showing  $\frac{60}{100} = \frac{30}{50} = \frac{12}{25}$  and  $\frac{60}{100} = 0.6$ , with a note "60%".
- Annotation 2:** A square with side length 5m. The student notes: "I know that  $\frac{60}{100} = 60\%$  because percentage is out of 100 and  $\frac{60}{100}$  is out of 100. So it is obvious that  $\frac{60}{100} = 0.6$ ." and "I know that  $\frac{60}{100} = 0.6$  because the line that separates the numerator and the denominator represents division so  $60 \div 100 = 0.6$ ." The student also says: "I used a square because I used a square because 3 it is enclosed and 2 it has all equal sides. So I chose five because  $5 \times 2 = 20$  and it is easy to figure out which equals to 12."
- Annotation 3:** A regular pentagon with side length 3m. The student notes: "I used a pentagon because the trading is  $\frac{3}{5}$  so there are five sides so there would be 3 shaded."
- Annotation 4:** A calculation showing  $60\%$  of 20 is 12.
- Annotation 5:** A calculation showing  $60 \div 100 = 0.6$ .
- Annotation 6:** A calculation showing  $60 \div 100 = 0.6$ .

#### Annotations

- 1 Fluency**  
Connects the concepts of percentage, fraction and decimal by accurately representing 60% as a fraction and a decimal and explains why the representations are equal
- 2 Problem-Solving**  
Chooses the side length of the square to be 5 metres, with the explanation that 'it is easy to figure out 60% of 20' following calculation of the perimeter of the square (20 metres)
- 3 Reasoning**  
Justifies the choice of a square as an appropriate shape for the area to be fenced
- 4 Understanding**  
Accurately calculates 60% of the perimeter of the fence (painted part) to be 12 (metres) and accurately represents the length of the painted part on a diagram representing the fence
- 5 Understanding**  
Recalls that the square and the regular pentagon are

shapes that have all of their sides of equal length

- 6 Reasoning**  
Justifies the choice of a regular pentagon as an alternative shape for the area to be fenced and correctly identifies the number of sides of the fence (3 of 5) that would be painted

## Measurement and geometry: Dimensions of a 3D object

### Sample summary

Students were asked to determine the dimensions of a box given the length of ribbon of 1.2 metres needed to wrap around the parcel.

### Proficiencies

#### Understanding

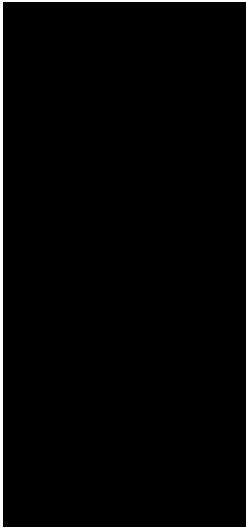
#### Fluency

#### Problem-Solving

#### Reasoning

At this year level understanding includes describing properties of different sets of numbers, using fractions and decimals to describe probabilities, representing fractions and decimals in various ways and describing connections between them, and making reasonable estimations.

### Worksheet

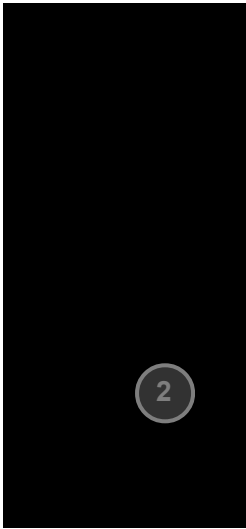
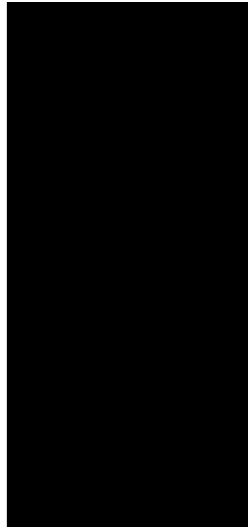


Jasmine has a parcel to be wrapped with ribbon. The length of the ribbon is 1.2m.  
The parcel is a rectangular prism.  
What could the size of the parcel be? Don't forget to include extra ribbon for the bow.

$1.2 = 120\text{ cm}$   
 $120 = 4L + 4W + 4H$   
 $30 = L + W + H$

L	21.6 cm	19.2 cm
W	4.8 cm	8.8 cm
H	4.8 cm	11.4 cm

$L = 21.6\text{ cm}$   
 $W = 4.8\text{ cm}$   
 $H = 4.8\text{ cm}$



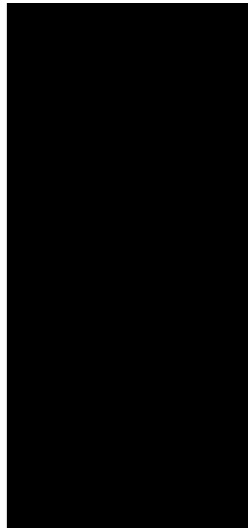
Jasmine has a parcel to be wrapped with ribbon. The length of the ribbon is 1.2m.  
The parcel is a rectangular prism.  
What could the size of the parcel be? Don't forget to include extra ribbon for the bow.

$L = 21.6\text{ cm}$   
 $W = 4.8\text{ cm}$   
 $H = 4.8\text{ cm}$

$4L + 4W + 4H = 1.2$   
 $4(21.6) + 4(4.8) + 4(4.8) = 1.2$   
 $86.4 + 19.2 + 19.2 = 1.2$   
 $124.8 = 1.2$

$124.8 - 1.2 = 123.6$   
 $123.6 / 4 = 30.9$

$L = 30.9 - 21.6 = 9.3$   
 $W = 30.9 - 4.8 = 26.1$   
 $H = 30.9 - 4.8 = 26.1$



## Annotations

- 1 Understanding**  
Represents the amount of ribbon (without bow) to be used to wrap the parcel as appropriate multiples of its length, breadth and height
- 2 Fluency**  
Calculates efficiently and accurately the amount of ribbon (without bow) required for wrapping the parcel and the amount of ribbon remaining for the bow, and describes the strategies used

## Student demonstration



## Number and algebra: Game show – Licence to solve

### Sample summary

Students participated in a dice rolling game where they rolled two number dice and an operation die to determine the mathematical operation to be performed. The winner was the student with the highest number.

### Proficiencies

#### Understanding

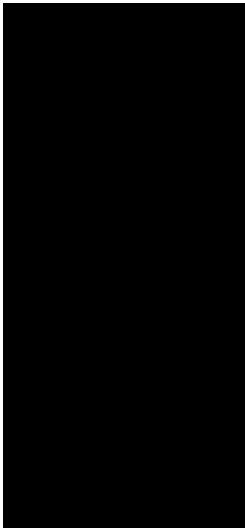
#### Fluency

#### Problem-Solving

#### Reasoning

At this year level understanding includes describing patterns in uses of indices with whole numbers, recognising equivalences between fractions, decimals, percentages and ratios, plotting points on the Cartesian plane, identifying angles formed by a transversal crossing a pair of lines, and connecting the laws and properties of numbers to algebraic terms and expressions.

Worksheet



**000 - Licence to solve**

Congratulations, you have been randomly selected to appear on the much loved game show 'Licence to solve'. The trick to winning this game is that you must use the following template to calculate your winnings:

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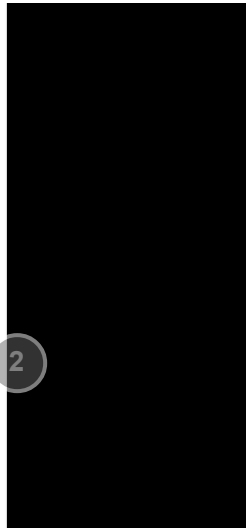
In pairs:

1. Each student takes turns at rolling the dice with numbers 1-6.
2. Using the template, you can choose which space to put the number rolled.
3. After the three number spaces are filled, you then use dice to roll your operation.
4. Again, you can choose which of the two locations to put this operation.
5. The student with the largest answer wins.

4 + 1 × 5 = 10	WIN	\$ 10	LOSS	\$ 0
6 ÷ 3 × 6 = 12	WIN	\$ 0	LOSS	\$ 7.33
6 × 4 + 3 = 27	WIN	\$ 0	LOSS	\$ 27
6 ÷ 2 + 3 = 6	WIN	\$ 6	LOSS	\$ 0
4 ÷ 4 × 2 = 2	WIN	\$ 4	LOSS	\$ 0
4 ÷ 2 - 2 = 1	WIN	\$ 0	LOSS	\$ 1
2 × 4 × 4 = 32	WIN	\$ 28	LOSS	\$ 0
3 - 3 ÷ 1 = 0	WIN	\$ 0	LOSS	\$ 0

Total winnings:

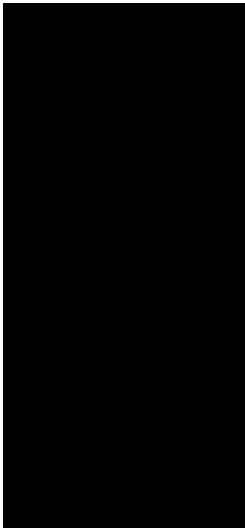
**\$ 82.67**



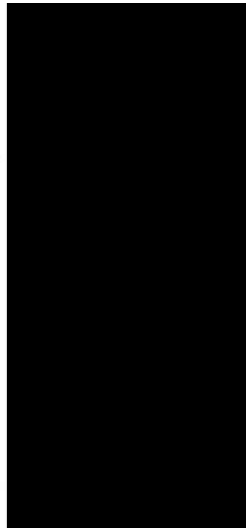
**Annotations**

**1 Understanding**  
In analysing and interpreting the problem, connects knowledge of the order of operations to develop a strategy for placing the numbers and operations rolled on the dice into the given template

**2 Fluency**  
Applies the order of operations appropriately in accurately calculating each of the different answers, and accurately determines the corresponding 'Win' or 'Loss' and the 'Total winnings'



*My strategy was to roll the highest numbers while still trying to roll addition and multiplication symbols. This didn't work in many ways so I didn't win.*



**Annotations**

**1 Reasoning**  
Describes the strategy used and provides a statement of justification for the strategy

**Number and algebra: Simplifying fractions**

**Sample summary**

Students were asked to solve a problem, where they had to determine the missing numerator in a fraction. They were given a set of clues to calculate the numerator.

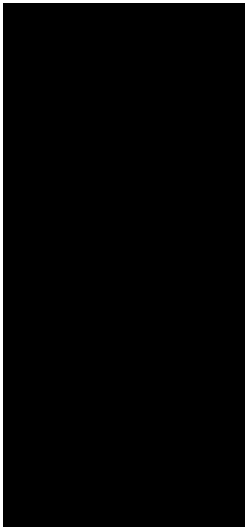
**Proficiencies**

**Understanding**

## Fluency

At this year level understanding includes describing patterns involving indices and recurring decimals, identifying commonalities between operations with algebra and arithmetic, connecting rules for linear relations with their graphs, explaining the purpose of statistical measures and explaining measurements of perimeter and area.

### Worksheet



Simplifying Fractions

3 + Numerator  
4 → Denominator  
Simplify Fraction =  $\frac{3}{4}$   $\frac{3}{4}$   $\frac{3}{4}$

It is three quarters

1.  $\frac{3}{4}$  2.  $\frac{3}{4}$  3.  $\frac{3}{4}$

If you half 2 and the half 4 you get  $\frac{3}{8}$   
Because what you do to the top you do to the bottom

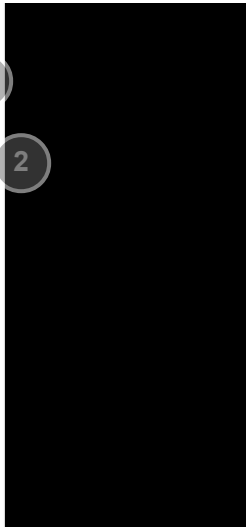
$6 \div 2 = \frac{3 \times 2}{10 \div 2} = \frac{6}{5}$

$\frac{3}{4} + \frac{4}{4} = \frac{7}{4}$  to prove fit 4 into 16  
 $16 \div 4 = 4$  0 times 4 into 16 4 times

$\frac{12}{10} = \frac{3 \times 4}{2 \times 5} = \frac{12}{10}$   
 $\frac{17}{30} = \frac{17 \times 2}{30 \times 2} = \frac{34}{60}$

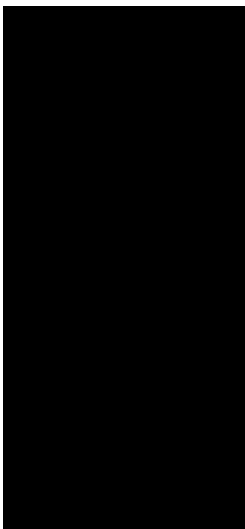
$\frac{48}{24} = \frac{48 \div 24}{24 \div 24} = \frac{2}{1}$   
 $\frac{12}{35} = \frac{12 \div 3}{35 \div 3} = \frac{4}{35}$

$\therefore 2 \times 2 \times 2 \times 3 = 48$   
 $\therefore 2 \times 2 \times 7 = 28$



### Annotations

- 1 **Fluency**  
Identifies different parts of the fractions
- 2 **Understanding**  
Describes operations on the fraction to create equivalent fractions
- 3 **Understanding**  
Reduces a number to the product of its prime factors



$\frac{24}{36} = \frac{2 \times 2 \times 2 \times 3}{2 \times 2 \times 2 \times 3}$  simplified to  $\frac{2}{3}$

$\frac{10}{32}$  simplified to  $\frac{5}{16}$

1.  $\frac{137}{250}$  Simplified form  $\frac{66}{125}$

2.  $\frac{128}{250}$

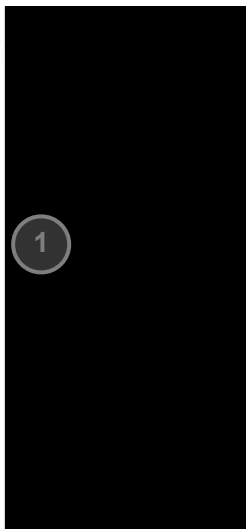
$\frac{132}{132} = \frac{2 \times 2 \times 3 \times 11}{2 \times 2 \times 3 \times 11} = 1$

$\frac{126}{126} = \frac{2 \times 3 \times 3 \times 7}{2 \times 3 \times 3 \times 7} = 1$

$\frac{18}{18} = \frac{2 \times 3 \times 3}{2 \times 3 \times 3} = 1$

$\frac{7}{7} = \frac{7}{7} = 1$

$\frac{1}{36} = \frac{1}{2^2 \times 3^2}$



### Annotations

- 1 **Understanding**  
Uses the reduction of the fraction to its prime factors to simplify the fraction

## Measurement and geometry: Equal areas

### Sample summary

Students were asked to justify the conditions for when a kite and a trapezium would have the same area.

### Proficiencies



# Understanding

## Fluency

## Problem-Solving

## Reasoning

At this year level understanding includes describing patterns involving indices and recurring decimals, identifying commonalities between operations with algebra and arithmetic, connecting rules for linear relations with their graphs, explaining the purpose of statistical measures and explaining measurements of perimeter and area.

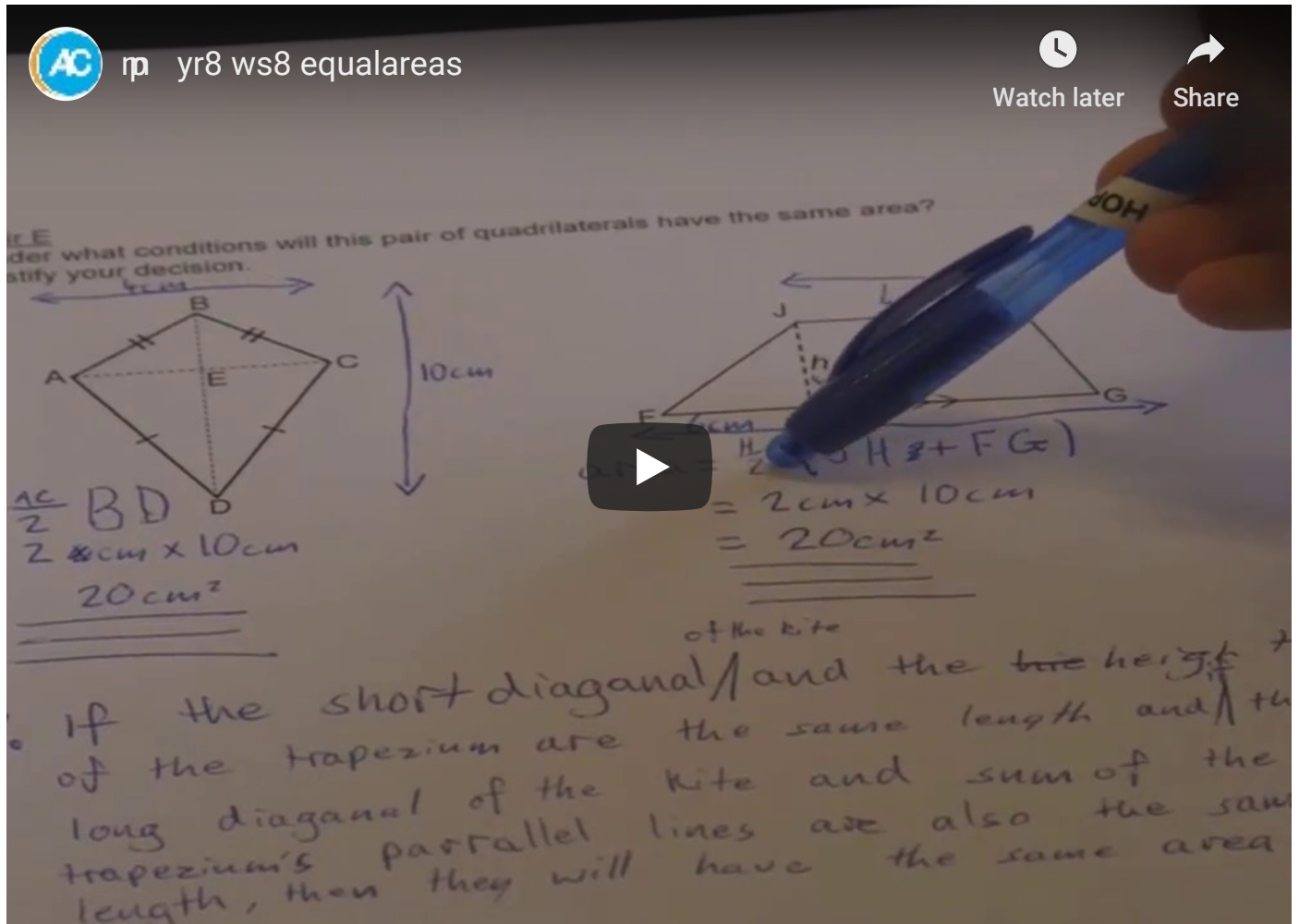
### Worksheet

### Annotations

- 1 Fluency**  
 Recalls the area formula for a kite and for a trapezium and expresses the formulas in relation to the given diagrams
- 2 Reasoning**  
 Investigates the conditions under which a kite and a trapezium will have the same area by comparing the components of the respective area formulas
- 3 Problem-Solving**  
 Deduces that if a kite and a trapezium are such that the length of one diagonal of the kite is equal to height of the trapezium then, for the areas to be equal, the length of the other diagonal of the kite must be the same as the sum of the lengths of the parallel sides of the trapezium
- 4 Understanding**  
 Chooses appropriate dimensions for a kite and a trapezium to demonstrate the truth of the conclusion reached

- 5 **Reasoning**  
Makes a statement that describes and generalises the condition for a kite and a trapezium to have the same area

**Student demonstration**



**Number and algebra: Archers in the plane**

**Sample summary**

Students were asked to find the coordinates of a point where an archery target could be placed such that it is equidistant from three archers.

**Proficiencies**

**Understanding**

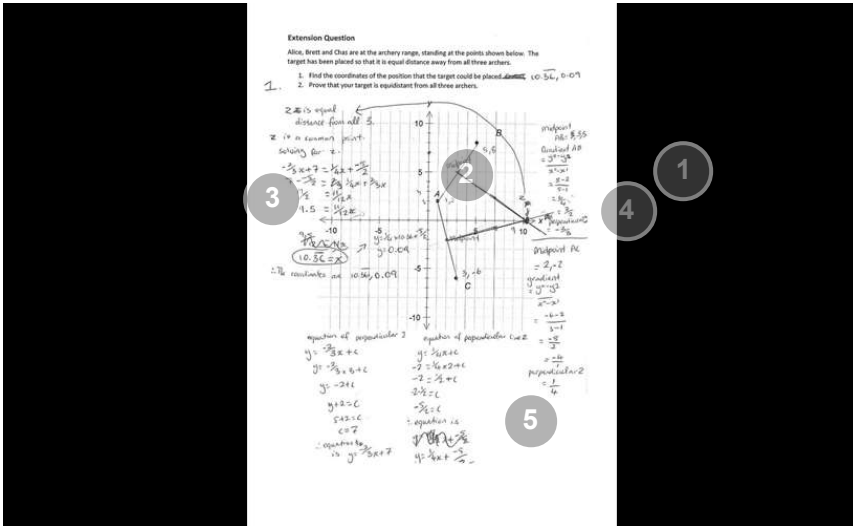
**Fluency**

## Problem-Solving

### Reasoning

At this year level understanding includes describing the relationship between graphs and equations, simplifying a range of algebraic expressions and explaining the use of relative frequencies to estimate probabilities and of the trigonometric ratios for right-angle triangles.

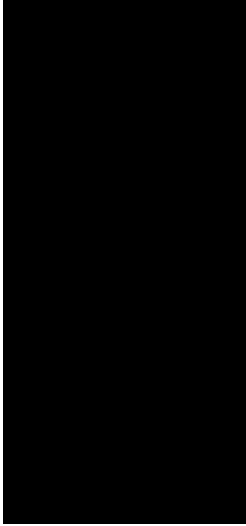
### Worksheet



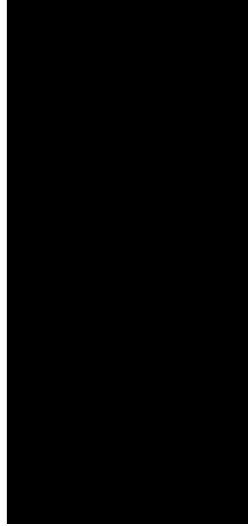
### Annotations

- 1 Fluency**  
 Calculates the midpoints and gradients of the intervals accurately and the gradients of the respective perpendicular bisectors of the intervals through the use of recalled formulas
- 2 Understanding**  
 Connects the relevant plane geometry and coordinate geometry concepts by identifying the need to determine the respective midpoints and gradients of the intervals
- 3 Understanding**  
 Identifies the need to determine the respective equations of the perpendicular bisectors of the intervals to determine the coordinates of the position in which the target should be placed
- 4 Understanding**  
 Interprets the problem as a geometric problem involving the intersection of the perpendicular bisectors of the intervals joining points A and B and points A and C
- 5 Problem-Solving**  
 Determines the equations of the perpendicular

bisectors accurately, solves the equations simultaneously to determine their point of intersection, and communicates the solution effectively



2<sup>nd</sup> Let  $T$  be the target and the three archers  $A, B, C$ . Then from our previous work, we know that  $T$  is the intersection of the perpendicular bisectors of  $AB$  and  $BC$ . We will show that  $T$  is also equidistant from  $A$  and  $C$ .  
 1. Consider the perpendicular bisector of  $AB$ . Let  $M$  be the midpoint of  $AB$ . Then  $TM \perp AB$ .  
 2. Similarly, let  $N$  be the midpoint of  $BC$ . Then  $TN \perp BC$ .  
 We will show that  $TA = TC$ .  
 Consider the right-angled triangles  $TMA$  and  $TNC$ . We have  $TM = TN$  (since  $T$  is on the perpendicular bisector of  $AB$  and  $BC$ ), and  $MA = NC$  (since  $M$  and  $N$  are midpoints of  $AB$  and  $BC$  respectively).  
 Therefore,  $\triangle TMA \cong \triangle TNC$  (RHS).  
 Hence,  $TA = TC$ .  
 This shows that  $T$  is equidistant from  $A$  and  $C$ .  
 Therefore,  $T$  is the target.



## Annotations

- 1 Understanding**  
 Connects congruency of triangles, Pythagoras' Theorem and distance on the Cartesian plane to establish a proof that the target has been located to be equidistant from all three archers
- 2 Reasoning**  
 Describes and justifies the strategy used to determine the point equidistant from all three archers

## Student demonstration

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## Statistics and probability: Baffling box plots

### Sample summary

From a given set of information about two footballers, students were asked to determine which footballer was more worthy of a reward. Students had to provide justification for their decision.

### Proficiencies

#### Understanding

#### Fluency

#### Problem-Solving

#### Reasoning

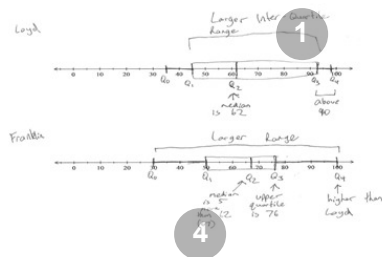
At this year level understanding includes applying the four operations to algebraic fractions, finding unknowns in formulas after substitution, making the connection between equations of relations and their graphs, comparing simple and compound interest in financial contexts and determining probabilities of two- and three-step experiments.

**Baffling Boxplots – Formative Assessment Task**

The Australian Football Legends Committee are deciding between two champion forwards to add to their hall of fame. They are going to decide between Essendon great Matthew Lloyd and Hawthorn/Sydney star Lance Franklin. They have been given the following data about their number of goals kicked per year to help them choose.

- Lloyd has kicked more than 90 goals in 25% of the years that he played, Franklin's upper quartile is 76 goals.
- Franklin's best season is better than Lloyd's best.
- Franklin's median number of goals kicked is 5 more than Lloyd's median of 62.
- Lloyd has a larger inter-quartile range, but a smaller range.

1. Draw the boxplot of the number of goals kicked per year for each player. Explain the choices you make for the location of the key points in each boxplot.



## Annotations

**1 Fluency**  
Represents a larger interquartile range and smaller range for Lloyd than for Franklin following appropriate choice of lower quartiles and minimum values

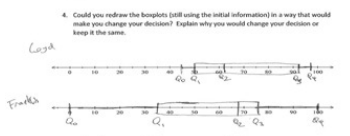
**2 Understanding**  
Chooses and represents appropriate upper quartile for Lloyd and appropriate maximum value ('best season') for Franklin that is higher than maximum value for Lloyd

**3 Understanding**  
Represents upper quartile for Franklin and represents median for Lloyd and for Franklin

**4 Reasoning**  
Explains location of medians, and upper quartile for Franklin, and chosen locations of upper quartile for Lloyd, maximum values, lower quartiles and minimum values through annotations on the boxplots

2. According to your boxplots, which of the two players was more consistent in their scores? Justify your choice.  
 Lloyd was more consistent because he has a smaller range. **1**

3. Explain, with reference to your boxplots, which of the players you would advise the committee to select into the Hall of Fame.  
 I would advise Franklin to the hall of fame because he has a higher median and max so he more often **2** scores higher than Lloyd.



Franklin looks a lot worse now because he is wildly inconsistent and his lower quartile to median is a lot lower.

5. If you were on the committee, would you be confident enough to make a decision based on this information? What other information would you ask for?  
 No, I would make for their mean score because that is a better representation of the players.

## Annotations

**1 Reasoning**  
 Compares the two boxplots and justifies the choice of more consistent player by referring to the difference in range

**2 Reasoning**  
 Compares the two boxplots and justifies the choice of inductee by referring to the difference in median and the difference in maximum value